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STRUCTURAL OPTIMIZATION OF A MATERIAL EXHIBITING NEGATIVE STIFFNESS

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Abstract. *In this paper a material realization of a structure exhibiting negative stiffness is presented. The structure is formed by polyethylene foam sheets that are used as a base material and the negative stiffness feature is provided by an inherent bistable element introduced with the help of curved cuts. The present material realization is intent to be optimized in order to supply improved passive damping performance in conformity with the literature, where it is demonstrated that extreme damping can be achieved in materials incorporating negative stiffness inclusions. It is shown that even for the initial guess, the damping of the structure is much superior than the one of the base material. Structural optimization can be performed numerically by the finite element method combined with some metaheuristic optimization algorithm. As design variables geometrical characteristics of the additional cuts and material density are considered. The objective function involves strain delay as a measure of the damping performance. As a necessary step, a reliable finite element model must be available. This paper shows material models testing and fitting of the material parameters according to the experimental examination. The difficulty is attributed to strong orthotropy and to the fact that during the displacement controlled test several regions of the structure are severely stretched or compressed and therefore the material model must be highly non-linear and cover correctly very large range of strains.*

1 INTRODUCTION

Significant progress in material processing has stimulated interest in material design optimization. New materials realizations can be tailored by optimization techniques to achieve optimal responses to given solicitations. It has already been proven that extreme damping can be attained in materials incorporating negative stiffness inclusions. Nevertheless, material realization in this regard is still a challenging task.

A typical representation of a negative stiffness mechanism is formed by two inclined springs/bars allowing for snap-though. Based on this feature and in continuation with previous works, Structure with inherent bistable elements (SIBE) periodically distributed is suggested to be made of polyethylene sheets with additional semi-circular cuts. Therefore, as a Base material (BM), polyethylene of low density (20-50kg/m^3) is used in form of plates made of two thinner sheets glued together, giving the total thickness of around 2 cm.

SIBE is unstable and therefore can only be examined under displacement control test. In preliminary testing it was shown [1] that SIBE provides the expected behaviour. The damping performance can be measured by the strain delay (imposed displacement) with respect to the generated stress (total force). It was shown that even the initial guess of SIBE without any optimization achieves damping characteristics much superior than the ones of the BM [2]. By techniques of structural optimization the global loss factor can be further improved and maximized. Several geometrical parameters describing the position and size of the additional cuts can be taken as design variables. As an additional parameter the material density can be introduced. The optimization can be accomplished within a finite element model and, at the same time, with the help of a simple discrete mechanism. The procedure of obtaining the discrete mechanism is described in [1] and it is reviewed at the end of this paper. The main advantage of this representation is that the mechanism dynamic behaviour characteristics can be solved analytically. Then the optimization can be easily accomplished.

For the structural optimization, based on the finite element analysis, an adequate material model accounting for orthotropy induced by the polyethylene bubbles direction, for high compressibility and stretching occurring close to the cuts and sharp corners, and for material viscoelasticity must be available. Several specimens of BM with different sizes and densities were tested experimentally under unilateral tension, compression, cyclic and relaxation tests using a universal testing machine. Distribution of the Lagrangian strain tensor was extracted with the help of digital image correlation and tracking and transformed to the left or right Cauchy-Green strain tensors. Material parameters of several hyperelastic models were adjusted by fitting modules, created in Matlab. Statistical treatment and design of experiments was used to identify significant factors and interactions among the size, shape and arrangement of the additional cuts, as well as the density of the BM. Allowable ranges of these factors define the design space and then the global loss factor can be maximized by some metaheuristic algorithm.

The paper is organized in the following way. In Section 2 resume of the most important experimental results is given, in Section 3 material fitting is presented and the global loss factors is extracted. In Section 4 the discrete model is described and the paper is concluded in Section 5.

2 EXPERIMENTAL EXAMINATION

Some characteristics of the BM and SIBE specimens of low density of 20kg/m^3 are presented in Table 1. These specimens were tested on a universal testing machine (model 5566, Instron Corporation Canton, USA) with a load cell of 10 kN. The cross-head velocity was in the range of 3 to 9 mm/min. At the time of these tests the digital image correlation and tracking equipment was not available.

	Width [mm]	Thickness [mm]	Length [mm]	Uniaxial Test
T01L	37.45	22.78	125	Tension (9mm/min)
T02T	39.92	22.38		
C01L	21.9	38.26	24.46	Compression (3mm/min)
C02T	21.93	38.63	24.45	
C03T	22.1	38.69	24.89	
C04L	25.14	39.1	24.97	
TS01	174	21.39	154	Tension (4mm/min)
TS02	174	21.39	154	Tension (9mm/min)

Table 1: Specimens characteristics.

Regarding the legend used in Table 1: first “T” means traction and “C” denotes compression. “01” and similar is the sample number. BM specimens have additional designation: “L” means that the stiffer direction was the longitudinal one, i.e. coincident with the direction of the applied force, and “T” that it was the transversal one. “S” in used for SIBE samples.

In Figure 1 resume of some results on BM is presented. It is seen that by separating the tension and compression tests, there is e non-smooth passage between positive and negative stresses which complicates the material fitting.

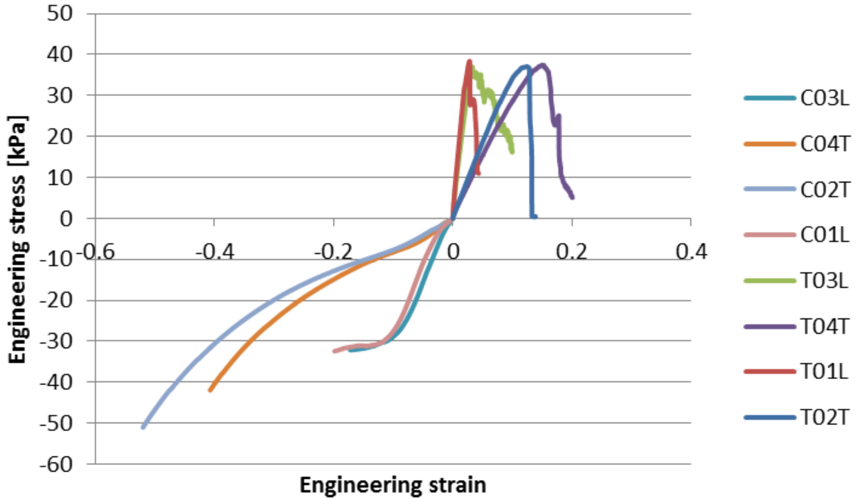

Figure 1: Resume of some results on BM.

Figure 2 presents an evolution of the tension test on SIBE. Figures 3 and 4 relate the imposed displacement with the total force. It is seen that the negative stiffness region is well-formed. In Figure 3 captions “-1” and “-2” stand for the number of the test performed on the same specimen TS02. Specimen TS01 was left until the failure. In addition, cyclic tests were performed on the specimen TS02. The objective of the cyclic tests was to show that in displacement controlled regime the specimen can operate solely over the negative stiffness branch. These results are presented in Figure 4.

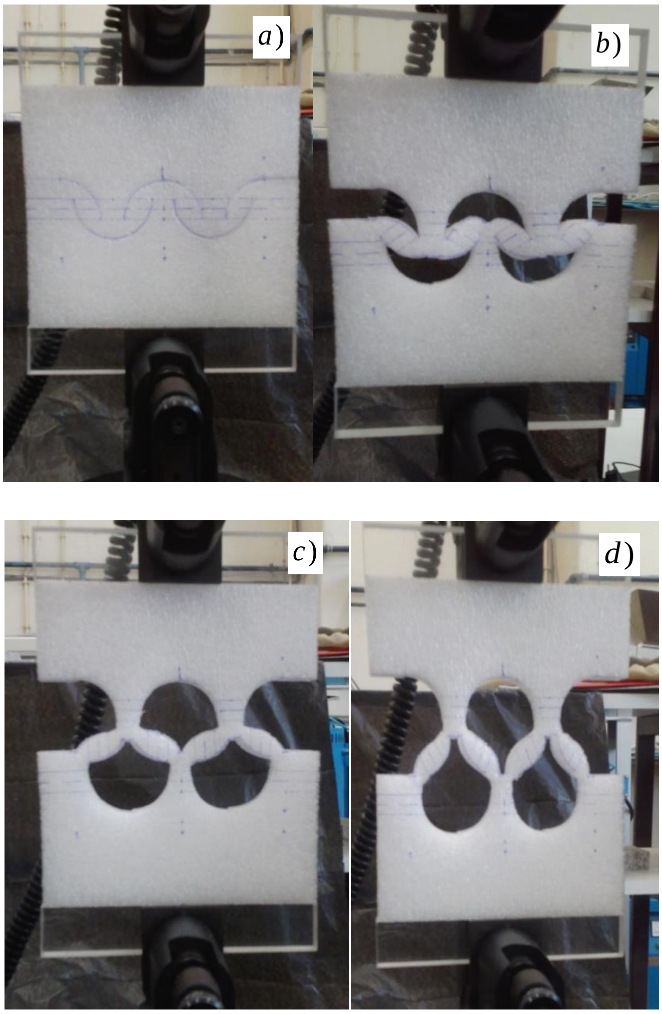

Figure 2: Tension test on the bistable material: (a) initial position, (b) beginning of the negative stiffness, (c) new beginning of the positive stiffness, (d) initiation of failure.

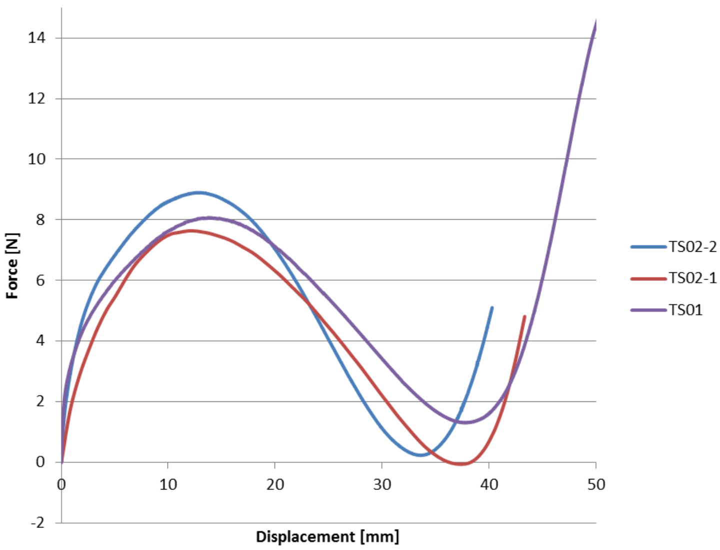

Figure 3: Tension tests on SIBE.

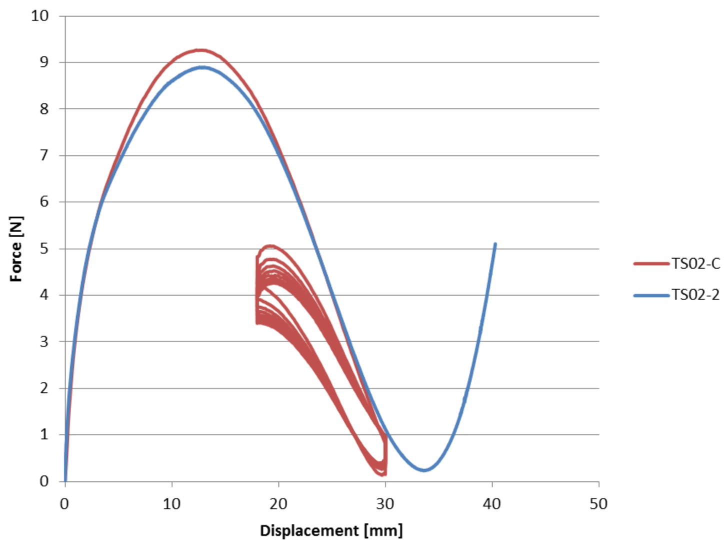

Figure 4: Tension and cyclic tests on SIBE.

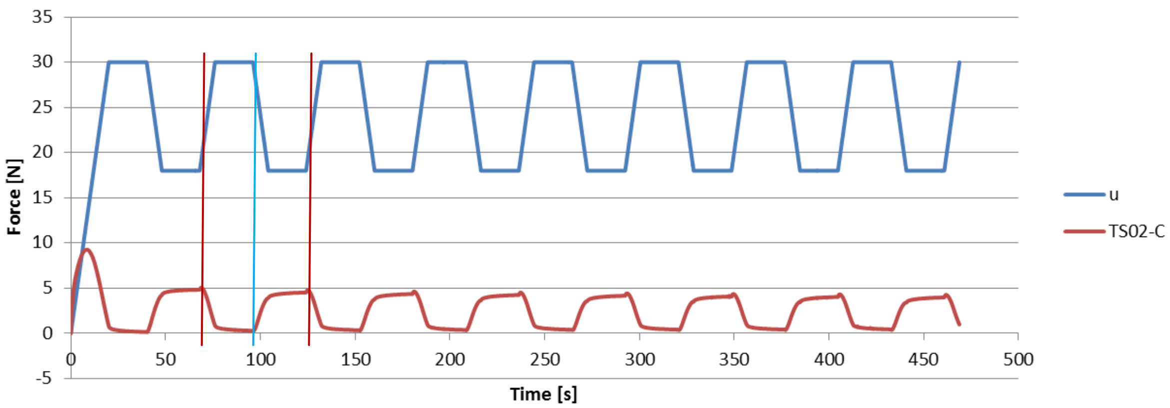

Figure 5: Damping estimation.

In Figure 5 the procedure for calculating the strain delay is briefly shown. Imposed displacement on SIBE (in mm) is compared with the registered force as a function of time. From the time delay $\tan\delta_{SIBE} = -0.32$ can be estimated. The same procedure, when applied on BM, gives $\tan\delta_{BM} = 0.07$, which demonstrates a significant improvement of SIBE versus BM.

3 MATERIAL FITTING

The experiments showed in previous section were described in terms of engineering (nominal) stress tensor and engineering strain. Therefore, the material models should be fitted in this form and not in terms of the true Cauchy stress. Several attempts summarized in [1] were performed on non-linear elastic materials in software ANSYS. Better results were, however, obtained on hyperelastic models, because of their numerical stability, as “infinite” compression is allowed without reverting the element volume to a negative value. There are few models that account for orthotropy, but acceptable results were obtained on isotropic highly compressible model specifically derived for compressible forms, i.e. the Ogden-Storakers model, with orthotropy simulated by additional constraints. The strain energy density with a suitable volumetric function is given by:

$$W = \sum_{k=1}^{n} 2\frac{\mu_k}{\alpha_k^2}\left[\lambda_1^{\alpha_k} + \lambda_2^{\alpha_k} + \lambda_3^{\alpha_k} - 3 + \frac{1}{\beta_k}\left(J^{-\alpha_k\beta_k} - 1\right)\right] \tag{1}$$

where α_k, β_k and μ_k are material constants, λ_i are principal stretches defined as square roots of eigenvalues of the Left Cauchy-Green strain tensor $\mathbf{B} = \mathbf{F}\cdot\mathbf{F}^T$, $\mathbf{F}$ is the deformation gradient and $J = \det(\mathbf{F})$. For the uniaxial tests, $\lambda_2 = \lambda_3$ and the longitudinal Cauchy stress is given by

$$\sigma_{11} = 2\left(\lambda_1\lambda_2^2\right)^{-1}\sum_{k=1}^{n}\frac{\mu_k}{\alpha_k}\left[\lambda_1^{\alpha_k} - \left(\lambda_1\lambda_2^2\right)^{-\alpha_k\beta_k}\right] \tag{2}$$

where the relation between λ_1 and λ_2 is obtained from the nullity of transversal stresses

$$0 = \sum_{k=1}^{n}\frac{\mu_k}{\alpha_k}\left[\lambda_2^{\alpha_k} - \left(\lambda_1\lambda_2^2\right)^{-\alpha_k\beta_k}\right] \tag{3}$$

For n=1 this relation can be explicitly written as

$$\lambda_2 = \lambda_1^{-\frac{\beta}{1+2\beta}} \tag{4}$$

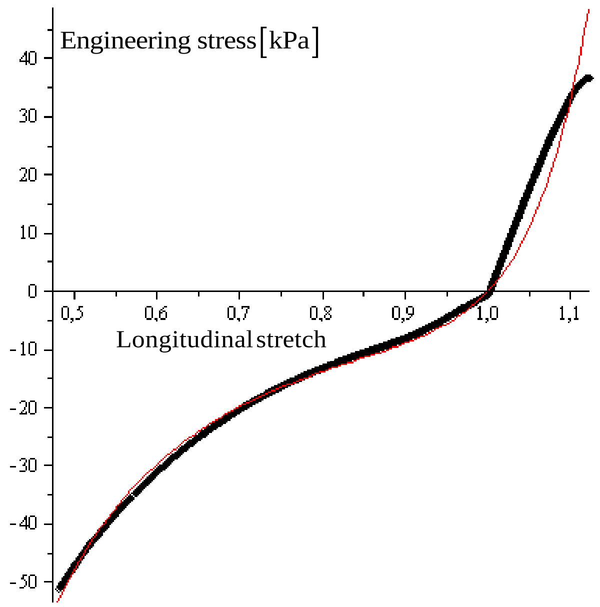

Figure 6: Fitting of the Ogden-Storakes model: black dots – experimental results, thin red line – analytical material curve.

For some experimental tests very good fitting could be obtained by MATLAB modulus, as shown in Figure 6. The analytical material curve is obtained for $\alpha = 17.9$, $\beta = 0.11$ and $\mu = 70\text{kPa}$. It is seen that the agreement is excellent especially in compression. Discrepancy in tension is caused by the discontinuity in slope of the experimental curve, originated by the fact that experiments in tension and compression were performed separately. Numerical results with the additional constraint are in good correlation with the experiment, as shown in Figure 7. For higher n, relation between stretches must be solved from a non-linear equation and due to a high number of material parameters, other optimization methods should be used.

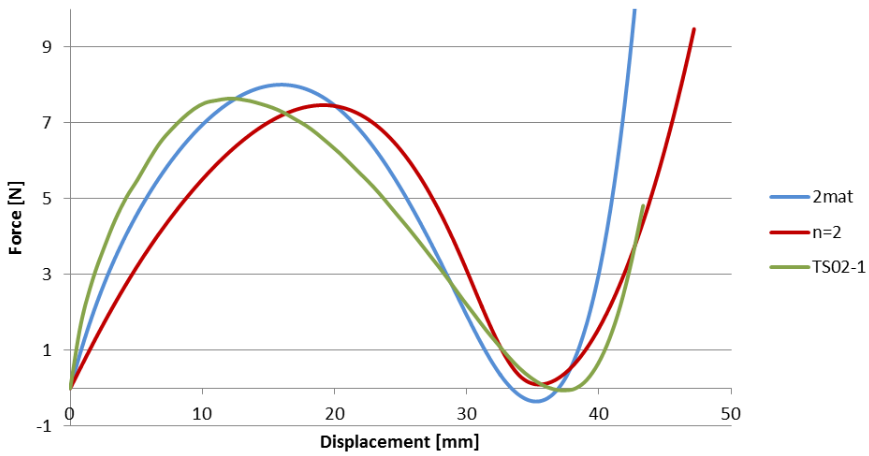

Figure 7: Results of finite element analysis: 2mat in blue stands for the fitted material with additional constraint, *n*=2 stands for the fitted material without the additional constraint.

4 DISCRETE MODEL

The discrete model can be formed by linear springs that geometrically represent SIBE as shown in Figure 8.

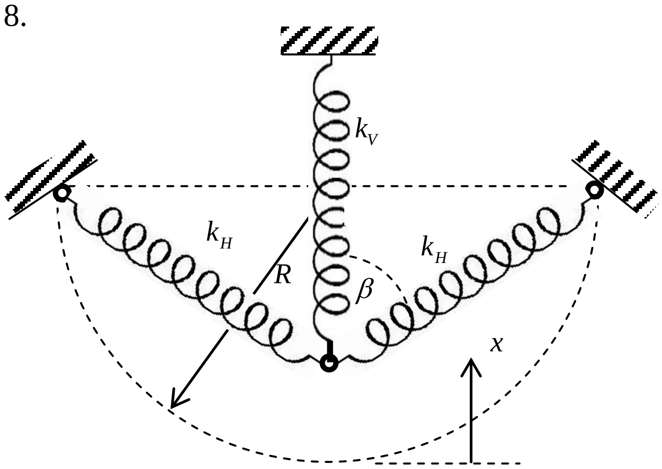

Figure 8: The discrete model.

Springs stiffnesses, k_V and k_H, can be optimized by implementation of the difference of the four fundamental conditions that express the position and value of the zero stiffness and can thus be represented analytically on the discrete model. In accordance with the experimental tests, it is assumed that the undeformed springs have their free extremities at the position $x=0$. Other geometrical details are related to the semi-circular cuts in the structure. The non-linear force representing the system in Figure 8 can be written as:

$$F_V = k_V x - 2\cos\beta N = k_V x - 2\frac{R-x}{\sqrt{R^2+(R-x)^2}} k_H \left(\sqrt{R^2+(R-x)^2} - R\sqrt{2}\right) =$$
$$k_V R\xi - 2(1-\xi) k_H R\left(1-\frac{1}{\sqrt{1-\xi+\xi^2/2}}\right) \tag{5}$$

where $\xi = x/R$ and N stands for the elastic force developed in the inclined springs. The objective is to tailor force-displacement curve of this model to be reasonably proximate to the physical model. The parameter connected to the specimen geometry, R, is equal to 25 mm. Consequently, there are only two variables, springs stiffnesses, and four essential conditions to fulfil. Optimal parameters, $k_V = 0.18\text{N/mm}$ and $k_H = 1\text{N/mm}$ can be obtained by a parametric optimization. As already stated, the discrete model can be easily analyzed. Stationary points are determined by:

$$\xi_1 = 1 - \frac{1}{k_V + 2k_H}\sqrt{-\left(k_V + 2k_H\right)^2 + 2\sqrt[3]{k_H^2\left(k_V + 2k_H\right)^4}} \tag{6}$$

$$\xi_2 = 1 + \frac{1}{k_V + 2k_H}\sqrt{-\left(k_V + 2k_H\right)^2 + 2\sqrt[3]{k_H^2\left(k_V + 2k_H\right)^4}} \tag{7}$$

Under the assumption that

$$8k_H\left(k_V + 2k_H\right)^2 \geq \left(k_V + 2k_H\right)^3 \tag{8}$$

The global performance of the discrete model is represented in Figure 8.

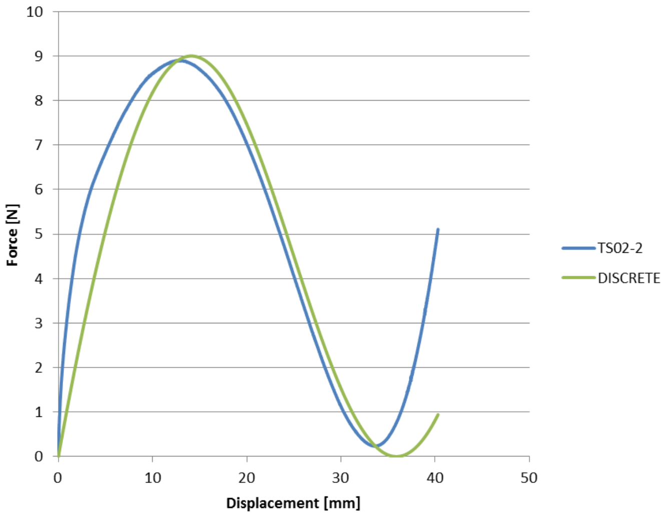

Figure 8: The discrete model performance versus the experiment.

It is seen that the fitted results looks quite good, nevertheless, the initial slope is very different from the tested specimen. It is seen, that a compressive force must be present, to secure the unstable horizontal position of the lateral springs, which is probably the effect of the testing machine. More tests and more details about the material behaviours must be gathered and analysed before the structural optimization can be started.

5 CONCLUSIONS

In this paper a material realization of a structure exhibiting negative stiffness, the structure with inherent bistable elements (SIBE), is presented. The present material realization can be optimized in order to supply extreme passive damping performance in conformity with the literature. It was shown that even for the initial guess, the damping of SIBE is much superior to the one of the base material (BM) formed by polyethylene foam sheets. Structural optimization can be performed numerically by the finite element method combined with some metaheuristic optimization algorithm. As design variables geometrical characteristics and material density of SIBE can be considered. The objective function involves strain delay as a measure of the damping performance. As a necessary step, a reliable finite element model must be available. This paper showed several attempts of reaching the suitable material model and procedures for material parameters fitting. The difficulty is attributed to strong orthotropy and to the fact that during the displacement controlled test several regions of the structure are severely stretched or compressed and therefore the material model must be highly non-linear and cover correctly very large range of strains. Nevertheless, acceptable correspondence with the finite element model and the discrete model has already been established.

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