A SEMI-ANALYTICAL APPROACH TO THE RELATIVE PERMEABILITY DETERMINATION IN LCM PROCESSES

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Liquid Composite Molding (LCM) processes inject resin into a stationary bed of fiber preforms to manufacture fiber reinforced composites. Infiltration of the resin into empty spaces between the fibers is driven by the hydrodynamic pressure gradient originated by the inlet pressure [1]. Sometimes, when the resistance of the preforms to resin flow is so high that in certain regions the hydrodynamic pressure can become very low. This causes the capillary pressure to exceed it and the driving mechanism of the flow will change. This is plausible when preforms consisting of fiber tows are used, where the spacing between the fiber tows is an order of magnitude higher than the spacing of pores inside the tows, forming dual porosity performs.

Little attention has been paid to unsaturated flows in dual scale porous media, where during infiltration a transition (partially filled) region is clearly visible, predicting that standard approaches to numerical simulation of filling phase using sharp resin front will not give satisfactory results. In order to describe the transition region, first of all, it is necessary to modify the macroscopic governing equations by introducing the relative permeability, k, and the macroscopic capillary pressure, P_c, as functions of the saturation, s:

$$\phi\frac{\partial s}{\partial t} = -\nabla\cdot\mathbf{v}^D(s),\ \mathbf{v}^D(s) = -\frac{k(s)}{\mu}\mathbf{K}\cdot\nabla\big(P(s)-P_c(s)\big).$$

$\mathbf{v}^D$ is called as the Darcy's velocity and $\mathbf{K}$, P, ϕ and μ are the absolute permeability tensor, the macroscopic pressure, the porosity of the preform, the resin viscosity, respectively, and t designates the time.

Unlike absolute permeability, no simple procedure is available to determine the relative permeability numerically. *A semi-analytical approach* is presented allowing its determination from microlevel analysis of *macroscopically one-dimensional flow*. Proposed approach is simple to use and it reveals that the relative permeability depends on both, surface tension as well as viscous force. Direct generalizations of the final result are also possible.

Microlevel analysis is based on Stokes law (**v** and p are the local velocity and pressure field):

$$\nabla\cdot\mathbf{v}=0,\ \nabla p=\mu\Delta\mathbf{v}$$

with the following boundary conditions (b.c.):

at the resin front: $\boldsymbol{\tau}^v\cdot\mathbf{n}=\mathbf{0}$ and $p=-2H\gamma$,

at the fiber boundary: **v**=**0**,

at the inlet: $\mathbf{v}=\mathbf{v}_0(t)$ or $p=p_0(t)$.

n is the unit normal to the resin surface, $\boldsymbol{\tau}^v$ is the viscous shear stress, H is the mean surface curvature and γ is the surface tension. Besides the b.c., the contact angle must be formed at the contact of the resin surface with the fiber. Resin progression is ensured by the free boundary condition $\partial f/\partial t+\mathbf{v}\cdot\nabla f=0$, where $f(\mathbf{x}(t),t)=0$ describes the position of the moving front.

The term *periodic solution* can be introduced for the solution (**v**, p) of steady state microlevel problem in fully saturated basic cell with b.c. stated as:

v fulfills periodicity b.c. on external cell boundaries,

v=**0** at the fiber boundary,

$\overline{p}=G\cdot\xi$ is prescribed on all boundaries.

$G<0$ is the imposed macrogradient and ξ is the spatial coordinate inside the cell in direction of the gradient. **v** is unique and p can be written as: $p=\overline{p}+\widetilde{p}+c$, where $\widetilde{p}$ is unique and fulfills periodicity b.c. on the external cell boundaries, $\overline{p}=G\cdot\xi$ inside the cell and c is constant. Viscosity enters the problem only as a linear analysis parameter.

Numerical results to support the semi-analytical approach were obtained by *free boundary program*. It uses the general-purpose finite element code Ansys and it is written in the Ansys Parametric Design Language and Fortran. Its initial form without surface tension influence was presented in [2]. The program permits to run transient microlevel problems and it is based on moving mesh scheme. The new resin front position is calculated using the free b.c. and explicit methods.

At an arbitrary flow front nodal point, the front is locally approximated by a smooth curve including two adjacent nodal points. Then the flow front can be described with respect to a local coordinate system by $x_n=g(x_t)$ and the free boundary condition takes form:

$$v_n - v_t\frac{\partial g}{\partial x_t} = v_n = \frac{\partial g}{\partial t}.$$

The new front position is given by:

$$x_{n,t_{k+1}} = x_{n,t_k} + (t_{k+1}-t_k)v_{n,t_k}.$$

The smooth curve approximation is repeated at the new front and the surface curvature is determined. Finally, Stokes problem is solved in the new domain.

Without surface tension effects the resin front progresses along the fiber boundary when other frontal point will touch it (Fig. 1a). If surface tension is included, the contact angle is adjusted to its given value by creating an additional curved surface (Fig. 1b).

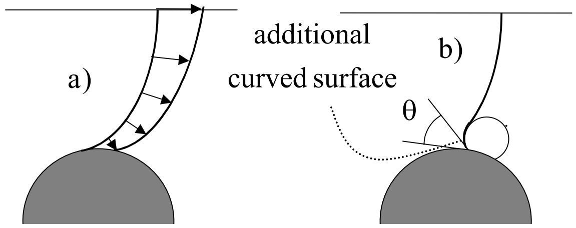

Fig. 1: Resin front progression.

In example of flow across an array of aligned cylindrical fibers different flow patterns are strongly related to the capillary number $N_{c,x} = v_x \mu / \gamma$. Flow progression for $N_{c,x} = \infty$ (bottom), 2.76, 0.276, 0.0276 and 0.00276 (top), respectively, is reported in Fig. 2.

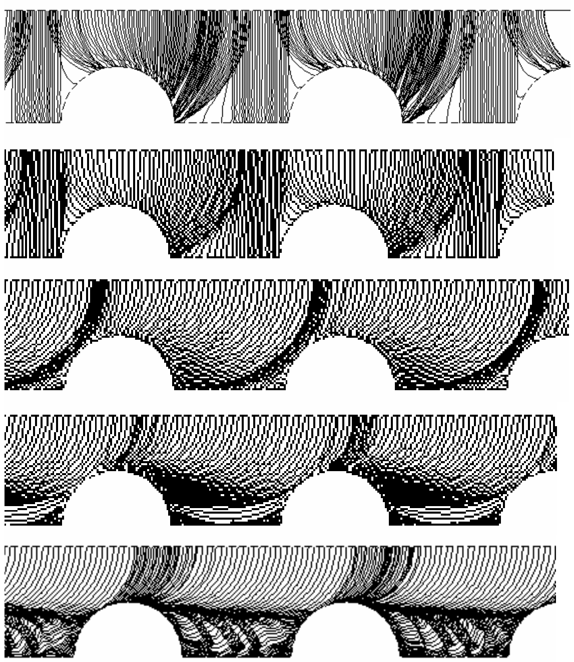

Fig. 2: Flow front progression

For the semi-analytical approach it is necessary to introduce additional terms. *The uniform basic cell*, is the basic cell, in which during the resin infiltration the saturation increases from 0 to 1, while the previous cell is fully saturated and the next cell is empty. The uniform cell in transition stage is called as *the transition cell*. After the transition cell is filled, it will take additional time until the distribution of **v** and p inside it will correspond to the periodic solution. We will call *uniform filling* a filling with constant flow rate, where (i) immediately after a transition cell is filled, **v** and p resemble periodic solution, (ii) phase averaged velocity is linear with respect to the spatial variable corresponding to resin front, ξ_s, and (iii) macropressure gradient is constant. In uniform filling the relative permeability is linear function of ξ_s.

The principal assumption in the semi-analytical approach is that after a transition cell is filled, (**v**, p) will resemble immediately the periodic solution. A reference cell must be chosen and ξ_{s_0} will mark a position of the uniform cell with respect to it.

The derivation of k_{xx} is fully analytical except of one particular point that requires numerical results. It proceeds in the following way. In filled cells it holds:

$$P^{ff}\left(\xi_s, \xi_{s_0}\right) = P^{fu}\left(\xi_s, \xi_{s_0}\right) + P^{v}\left(\xi_s, \xi_{s_0}\right) + A\left(\xi_{s_0}\right),$$

where $P^{v}\left(\xi_s, \xi_{s_0}\right)$ is intrinsic phase averaged pressure variable part fulfilling $P^{v}\left(0, \xi_{s_0}\right) = P^{v}\left(1, \xi_{s_0}\right) = 0$, $P^{fu}\left(\xi_s, \xi_{s_0}\right)$ corresponds to uniform filling, $A\left(\xi_{s_0}\right) - G/2$ is the initial value for $\xi_s = 0$ and $G < 0$ is the periodic pressure gradient. $P^{v}\left(\xi_s, \xi_{s_0}\right)$ reads as:

$$P^{v}\left(\xi_s, \xi_{s_0}\right) = \tilde{P}\left(\xi_s + \xi_{s_0}\right) - \tilde{P}\left(\xi_{s_0}\right) \qquad \text{for } \xi_s + \xi_{s_0} \le 1,$$

$$P^{v}\left(\xi_s, \xi_{s_0}\right) = \tilde{P}\left(\xi_s + \xi_{s_0} - 1\right) - \tilde{P}\left(\xi_{s_0}\right) \qquad \text{for } \xi_s + \xi_{s_0} > 1.$$

Other variables are expressed in similar way and average over ξ_{s_0} is done in order to ensure k_{xx} being independent on the uniform cell geometry. Finally:

$$k_{xx} = \frac{G \cdot \varepsilon_s}{G + \frac{2A_{P_c,\varepsilon_s}}{\varepsilon_s}\left(1 - \left(2\varepsilon_s - 1\right)^m\right)}$$

where A_{P_c,ε_s} is the mean value of the capillary pressure and ε_s is dimensionless counterpart of ξ_s. Graph of k_{xx} for flow across an array of aligned cylindrical fibers with $N_{c,x} = 0.166$ is shown below (Fig. 3).

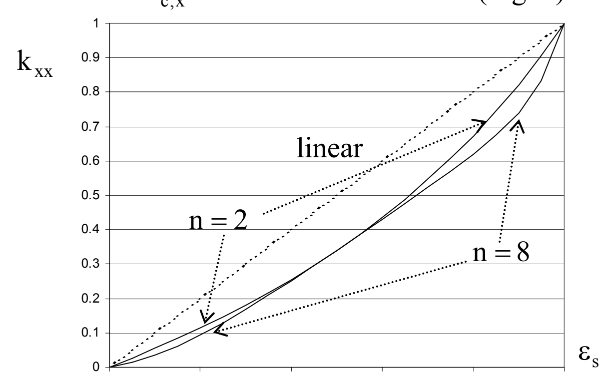

Fig. 3: Relative permeability as function of ε_s.

References:

[1] Advani, S.G., Bruschke, M.V., Parnas, R.S., Resin transfer molding, in *Flow and Rheology in Polymeric Composites Manufacturing*, Ed. S.G. Advani, Elsevier Publishers, Amsterdam, pp. 465-516, 1994.

[2] Dimitrovova, Z. and Faria, L., Finite element modeling of the RTM process, Proc. *5th International Conference on Flow Processes in Composite Materials*, July, Plymouth, UK, pp. 125-135, 1999.