Dynamic analysis of rail track for high speed trains. 2D approach

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ABSTRACT: In the framework of an ongoing national research project involving the University of Minho, the National Laboratory of Civil Engineering and the New University of Lisbon, different commercial FEM codes (DIANA, PLAXIS and ANSYS) were tested to model the dynamic performance of a high speed train track. Initially a plain strain model is considered in a robin test using experimental data from a well documented instrumented standard ballast rail track under the passage of a HST at 314 km/h.

Numerically predicted results are presented and assessed, and comparison is made between the different codes and also the experimental data.

1 INTRODUCTION

The behaviour of the railway track and infrastructure under the combination of high speed and repetitive axle loadings is affected as a result of a complex soil-structure interaction problem that constitutes a motive of geotechnical and structural R&D. In fact, high speed trains bring some new problems: (1) high train speeds demand tighter tolerances and track alignment for the purpose of safe and passenger comfort; (2) after a critical speed drastic dynamic amplification appears on the deformation of the track, embankment and supporting soft soil. Proper modelling of the dynamic behaviour of the railway track system, the soil and embankment materials and of the loading is essential to obtain realistic results. Additionally, measurements on actual railway sections is necessary for monitoring of the physical behaviour of the rail track and infrastructure for the calibration of the tools. These aspects are being investigated throughout an ongoing national project financed by the Foundation for Science and Technology involving the University of Minho (UM), the National Laboratory of Civil Engineering (LNEC) and the New University of Lisbon (UNL).

This paper presents preliminary results obtained with modelling using different numerical tools available at the different institutions: DIANA (UM), PLAXIS (LNEC) and ANSYS (UNL). A case study of an instrumented section of a high speed line was utilized using monitoring results presented by (Degrande & Schillemans, 2001), in order to evaluate the performance predicted based on the models created with the abovementioned tools..

In these early studies of comparative suitability of these tools to be used in pratical applications a 2D model was assumed for the analysis. As to whether 2D models are suitable for these analysis, there's an almost unanimous agreement between researchers that while they provide some understanding of the problem, 3D models are essential to reach accurate results. Some authors consider a special symmetry, which they call 2.5D. Another interesting simplified approach was proposed by Gardien and Stuit (2003) studying the modelling of soil vibrations from railway tunnels. These authors, instead of creating a three-dimensional model for the dynamic analysis built three complementary models: the first one is three-dimensional, where static loads were applied to obtain equivalent Timoshenko beam parameters, which are used in the second model to calculate the force time history under the sleeper; this force is then introduced in a plain strain model of the tunnel cross section. This type of approach could be interesting to develop for rail-track-foundation modelling.

Anyway, the reliability of all these numerical models depends largely on the accuracy of the input data and the choice of an appropriate underlying theory and can be evaluated through comparison with results from experiments and theoretical analysis. In this respect the results presented in this paper are a first contribution of the projectfor this assessment.

2 MODELLING OF DYNAMIC PERFORMANCE OF RAIL TRACK UNDER HIGH SPEED MOVING LOADS

2.1 *Background*

In the process of modeling and design, material models are an important component. Associated with the material models, it is necessary to take into account the tests needed to obtain their parameters. This chain of operations must be always borne in mind at the design stage in order to ensure a good planning of the tests needed for the models. This process will be completed by choosing the performance models and relevant design criteria (Gomes Correia, 2001, 2005).

For a more practical application, the complexity of all the process is divided in three levels: routine design, advanced design and research based design. An outline of these levels of design were reported in COST 337, action related with pavements and summarized by Gomes Correia (2001). Each process should be object of verification, calibration and validation. Verification is intended to determine whether the operational tools correctly represent the conceptual model that has been formulated. This process is carried out at the model development stage. Calibration refers to the mathematical process by which the differences between observed and predicted results are reduced to a minimum. In this process parameters or coefficients are chosen to ensure that the predicted responses are as close as possible to the observed responses. The final process is validation that ensures the accuracy of the design method. This is generally done using historical input data and by comparing the predicted performance of the model to the observed performance.

Based in this general framework, some particular aspects are developed hereafter in relation with the use of different numerical tools that are available for be used at routine and advanced modeling and design.

Lord (1999) emphasized the empirical rules still used at the construction and design levels of rail track. He also noticed the importance to consider dynamic aspects in design.

Rail track design is probably one of the most complex soil-structure interaction problems to analyse. The various elements in design process comprise (Lord, 1999): (1) multi-axle loading varying in magnitude and frequency; (2) deformable rails attached to deformable sleepers with flexible fixings, with sleeper spacings which can be varied; (3) properties and thickness of ballast, sub-ballast, prepared subgrade (if adopted); (4) properties of underlying soil subgrade layers.

At routine design level, several railway track models are operational and some commercially available. The most popular and simplest model for rail track design represents the rail as a beam, with concentrated wheel loads, supported by an elastic foundation. The stiffness of elastic foundation incorporates the sleepers, ballast, sub-ballast and subgrade, but it is not possible to distinguish between the contribution of the sleeper and underlying layers. This simplified approach has been used to establish dimensionless diagrams in order to quickly assess the maximum track reactions both for a single axle load and a double axle load when the track parameters are changed (Skoglund, 2002).

More sophisticated models have been developed which represent the rails and sleepers as beams resting on a multiple layer system (as in pavements) comprised of the ballast, sub-ballast and subgrade. These models include the commercial programs ILLITRACK, GEOTRACK and KENTRACK cited by Lord (1999).

In these models incorporating multiple layer systems, the design criterion is identical as for pavements, keeping vertical strain or vertical stress at the top of subgrade soil below a determined limit. This criterion is an indirect verification of limited permanent settlements at the top of the system, having the same drawbacks as mentioned for pavements. Gomes Correia & Lacasse (2005) summarized some values of allowed permanent deformations for rail track adopted in some countries.

To overcome the drawbacks of the previous models, two directions of advanced modelling are identified. The first category of models aim at improving the theory of beam resting on continuous medium by introducing a spring-dashpot to better simulate a multiple layer system. Furthermore, the model was also improved by introducing a moving load at constant speed and also an axial beam force (Koft and Adam, 2005). Figure 1 is a sketch of the model. This model is able to determine dynamic response in different rail track systems due to a load moving with constant speed. A drawback of the model is that it is limited to beams with finite length and consequently only steady state solutions can be provided.

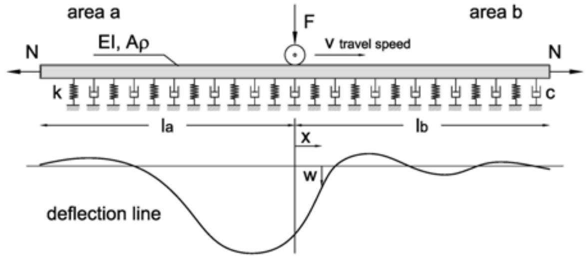


Figure 1. Sketch of a flexible beam resting on continuous spring-dashpot elements loaded by a moving single load (Koft and Adam. 2005).

The second group of advanced rail track models use FEM and hybrid methods. The hybrid methods couple FEM and multi-layer systems (Aubry et al., 1999; Madshus, 2001). The track-embankment system is modelled by FEM and the layered ground through discrete Green's functions (Kaynia et al., 2000). The software developed is called VibTrain. The models referred by Aubry et al. (1999) and Madshus (2001) use frequency domain analysis having the drawback to require linear behaviour of materials. However, solutions in the time domain also exist (Hall, 2000 – mentioned by Madshus, 2001).

This last family of models incorporating track-embankment-ground is very powerful as it simulates behaviour at all speeds from low up to the critical speed.

As referred by Madshus (2001), these models need further validation by field monitoring. Dynamic materials characterization should be strongly encouraged, as well as dynamic field observations, mainly for ballasts.

In this paper the second group of advanced models was tested for a case study. These first results only address calculations done in plain strain conditions (2D).

This project also intends to put into operation a numerical model where, by incorporating the global behaviour of the whole of the railway platform and the supporting soil, will serve to quantify advantages and disadvantages, of the methods used at present in the maintenance of ballast platforms. It will be also possible to quantify and predict the consequences that can have, on this type of structures, the increase of the circulation velocity and the axle load in high speed trains.

2.2 *Application of different commercial software for a case study*

2.2.1 *Presentation of the case study*

The experimental data used to calibrate de models is obtained from the literature (Degrande & Schillemans, 2001) with material parameters summarized in Table1.

These data correspond to vibration measurements made during the passage of a Thalys (high speed train – HST) at 314 km/h on a track between Brussels and Paris, more precisely near Ath, 55 km south of Brussels. The geometry and load characteristics of the HST are presented in Figure 2.

The HST track is a classical ballast track with continuously welded UIC 60 rails fixed with a Pandroll E2039 rail fixing system on precast, prestressed concrete monoblock sleepers of length l = 2.5m, width b = 0.285 m, height h = 0.205m (under the rail) and mass 300 kg. Flexible rail pads with thickness t = 0.01m and a static stiffness of about 100 MN/m, for a load varying between 15 and 90 kN, are under the rail.

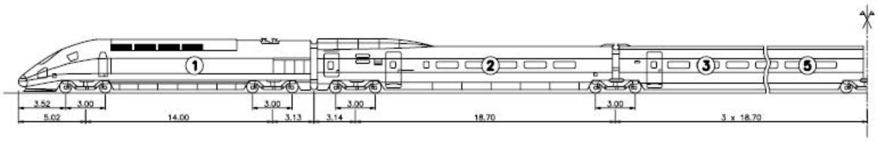
The track is supported by ballast and sub-ballast layers, a capping layer and the supporting soil.

Table 1. Geometry and material parameters of the HST track (after Degrande & Schillemans, 2001)

Element	Parameter	Value
Sleeper	Poisson	0.2 *
	Young Modulus	30 GPa *
	Mass density	2054 Kg/m^3
Rail/sleeper interface	Thickness	0.01 m
	Stiffness	100 MN/m
Rail (UIC60)	Area	76.84 cm^2
	Inertia I_x	3055 cm^4
	Inertia I_z	512.9 cm^4
	Tortional inertia	100 cm^4 *
	Volumic Weight	7800 Kg/m^3
	Poisson	0.3 *
	Young Modulus	210 GPa *
Ballast (25/50)	Stiffness	0.3 m
	Mass density	1800 Kg/m^3 *
	Poisson	0.1 *
	Young Modulus	200 MPa *
	Damping	0.01 *
Sub-ballast (0/32)	Stiffness	0.2 m
	Mass density	2200 Kg/m^3 *
	Poisson	0.2 *
	Young Modulus	300 MPa *
	Damping	0.01 *
Capping layer (0/80 a 0/120)	Stiffness	0.5 m
	Mass density	2200 Kg/m^3 *
	Poisson	0.2 *
	Young Modulus	400 MPa *
	Damping	0.01 *
Soil 1	Compression wave velocity	187 m/s
	Mass density	1850 Kg/m^3
	Stiffness	1.4 m
	Shear wave velocity	100 m/s
	Poisson	0.3
	Damping	0.03
Soil 2	Compression wave velocity	249 m/s
	Mass density	1850 Kg/m^3
	Stiffness	1.9 m
	Shear wave velocity	133 m/s
	Poisson	0.3
	Damping	0.03
Soil 3	Compression wave velocity	423 m/s
	Mass density	1850 Kg/m^3
	Stiffness	Infinite
	Shear wave velocity	226 m/s
	Poisson	0.3
	Damping	0.03

* Adopted values

Figure 2 shows the configuration of the Thalys HST referred by Degrande & *Lombaert* (2000), consisting of 2 locomotives and 8 carriages; the total length of the train is equal to 200.18 m. The locomotives are supported by 2 bogies and have 4 axles. The carriages next to the locomotives share one bogie with the neighbouring carriage, while the 6 other carriages share both bogies with neighbouring carriages. The total number of bogies equals 13 and, consequently, the number of axles on the train is 26. The carriage length Lt, the distance Lb between bogies, the axle distance La and the total axle mass Mt of all carriages are summarized.



	# carriages	# axles	L_t [m]	L_b [m]	L_a [m]	M_t [kg]
Locomotives	2	4	22.15	14.00	3.00	17000
Side carriages	2	3	21.84	18.70	3.00	14500
Central carriages	6	2	18.70	18.70	3.00	17000

Figure 2. Geometry and load characteristics of the Thalys HST (Degrande & *Lombaert,* 2000)

The location of the measurement points (accelerometers) used for this work is presented in Figure 3 and the results are shown hereafter together with numerical predictions.

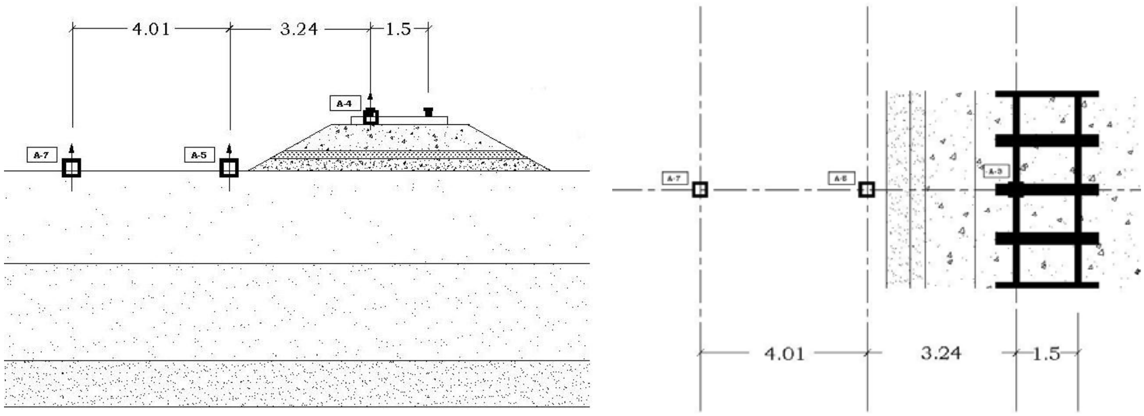


Figure 3. Location of the accelerometers

2.2.2 *Simulation of Thalys HST moving at 314 km/h*

The loads to be considered in the calculations are those that derive from the passing of the train. The loading history depends, naturaly, on the train speed. Because the calculations are to be done in plain strain conditions places some difficulties in the definition of the loading model arise because any point load in a plain strain model corresponds to a distributed load in the tree-dimensional model. Therefore, it is necessary to consider some simplifications and assumptions and, in the interpretation of the results, to limit its validity to the distances reported in Gutowski & Dym (1976). The maximum length of the Thalys train is L_T=196.7 m (maximum distance between extremity axles) so, the results obtained in 2D model considering a linear load can be considered as valid for distances:

$$d = \frac{L_T}{\pi} = \frac{196.7}{\pi} \approx 62\,m \quad (1)$$

Although the loads transmitted by the axle are discrete, the stiffness of the structural elements of the railroad superstructure provides some distribution of the load. For circulation speeds – V – lower than the critical speed - Vcr - the loading in each point due to the passage of a axle follows approximately the distribution presented in Figure 4. The exact shape of the curve depends on the load speed, on the response of the railroad superstructure and its foundation, being that for higher speeds (but still inferior to the critical one) the curve tends to be thinner. When the speed approaches the critical speed, the curve loses the symmetry and the maximum value occurs after the load.

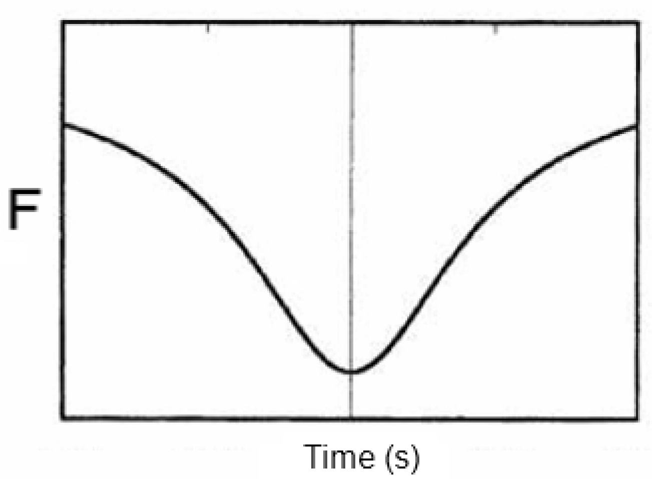


Figure 4. Load due to a axle at sub-critical speed

A possible approach to define the load distribution can be considered admitting a distribution adjusted to the sleeper spacing as is considered in the Japanese regulations. In accordance with this document, a changeable part between 40 and 60% of the load is distributed to the adjacent sleepers.

A simplified way to establish the load distribution consists of using the solution of the Winkler beam for the movement of a load. In accordance with this simplified model, the quasi static response in displacement is given by:

$$w(s)=\frac{Q}{2kL}e^{-|s/L|}\left(\cos\left|\frac{s}{L}\right|+\sin\left|\frac{s}{L}\right|\right) \tag{2}$$

where Q if the applied load, k the reaction module of the foundation, L the characteristic length of the beam and s the coordinate in a moving referential.

It seems reasonable to admit that, for speeds inferior to the critical one, the distribution of load underneath each axle will follow an analogous distribution:

$$F(s)=\frac{F_e}{2L}e^{-|s/L|}\left(\cos\left|\frac{s}{L}\right|+\sin\left|\frac{s}{L}\right|\right) \tag{3}$$

In the previous equation F(s) represents the distribution of the force due to each axle as a function of force F_e correspondent to the axle. The value of characteristic length L can be adjusted to obtain a certain amount of axle load at the point s=0 (underneath the axle).

Transformation between static referential “x” (in global coordinates) and the moving referential “s” is obtained through:

$$s=\frac{1}{L}(x-V_0t) \tag{4}$$

where V_0 represents the train speed and t the time.

Thus, admitting that for s=0 one has 60% of the axle load, L=0.831 will be obtained. The load distribution corresponding to each axle is presented in Figure 5 for a unitary load.

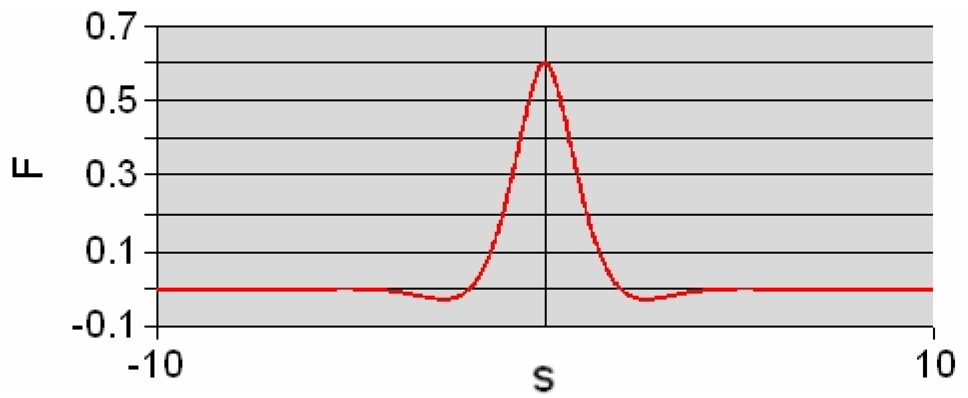


Figure 5. Load distribution to a unitary axle load

The effect of the train can now be obtained considering the overlapping of all the axles, in accordance with the load distribution of the Thalys train:

$$F = \sum_{i=1}^{n} F_i \qquad (5)$$

At the speed of 314 km/h (87.22 (2) m/s), the train passes in each section in 2.255 s. The modelling must start a little before the first axle passes in the calculation section because its effect starts before the load passes in that section. The computation must end some time later, in order to stabilize the vibrations in the surrounding media.

Combining the diverse axles in accordance with the geometry and load distribuition of the train and applying the previous expressions, it is possible to establish the loading plan to apply. This plan is represented in Figure 6.

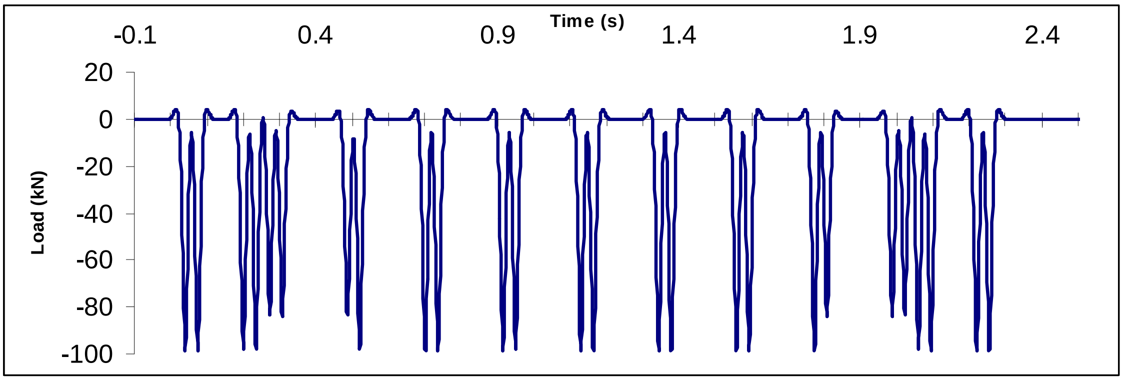


Figure 6. Loading plan for the Thalys HST at V=314 km/h

2.2.3 *First set of results of accelerations at different observation points*

Different commercial softwares were used by different institutions for modelling the behaviour of high-speed rail tracks.

The University of Minho team uses DIANA software. This program is a general finite element code based on the Displacement Method (DIANA = DIsplacement method ANAlyser). It features extensive material, element and procedure libraries based on advanced database techniques.

The models created with this software are 63.3m width and 65m height, given the symmetry of its geometry and loading, only half of the track was modelled.

The final models comprised 2775 elements, being 12 triangular elements of 6 nodes (named CT12E in DIANA), 2601 quadrilateral elements of 8 nodes (Q8EPS), and 162 bounding elements (L4TB). The bounding elements were used at the limits of the model to take into account the propagation of waves to outer regions. Figure 7a shows the mesh used for numerical simulations.

The material properties are those defined in Table 1. Furthermore, DIANA considers Rayleigh damping according to:

$$[C]=\alpha[M]+\beta[K] \tag{6}$$

where [C] represents the damping matrix [M] the mass matrix and [K] stiffness matrix. The parameters α and β are the damping coefficients.

In order to establish α and β, it is necessary to relate these parameters with the hysteretic damping more adequate to represent soils damping. This can be achieved relating the hysteretic damping coefficient (ξ) to α and β, for two known frequencies. In a separate modal analysis the first two frequencies obtained where (1.71 and 2.55 Hz). Using these frequencies and considering:

$$\xi_i = \frac{1}{2}\left(\frac{\alpha}{\omega_i} + \beta\omega_i\right) \tag{7}$$

Table 2 is obtained.

Table 2. Damping coefficients for DIANA and Plaxis models

Material	ξ	α		β	
		DIANA	Plaxis	DIANA	Plaxis
Ballast, sub-ballast and subgrade	0.01	0.128702	0.290113	0.000747	0.000342
Foundation	0.03	0.386105	0.870339	0.002242	0.001027

The LNEC team decided to use Plaxis Dynamic to perform the numerical simulations. Plaxis Dynamic 8.2, produced by Plaxis BV, Holland, is specially oriented to deal with geotechnical structures.

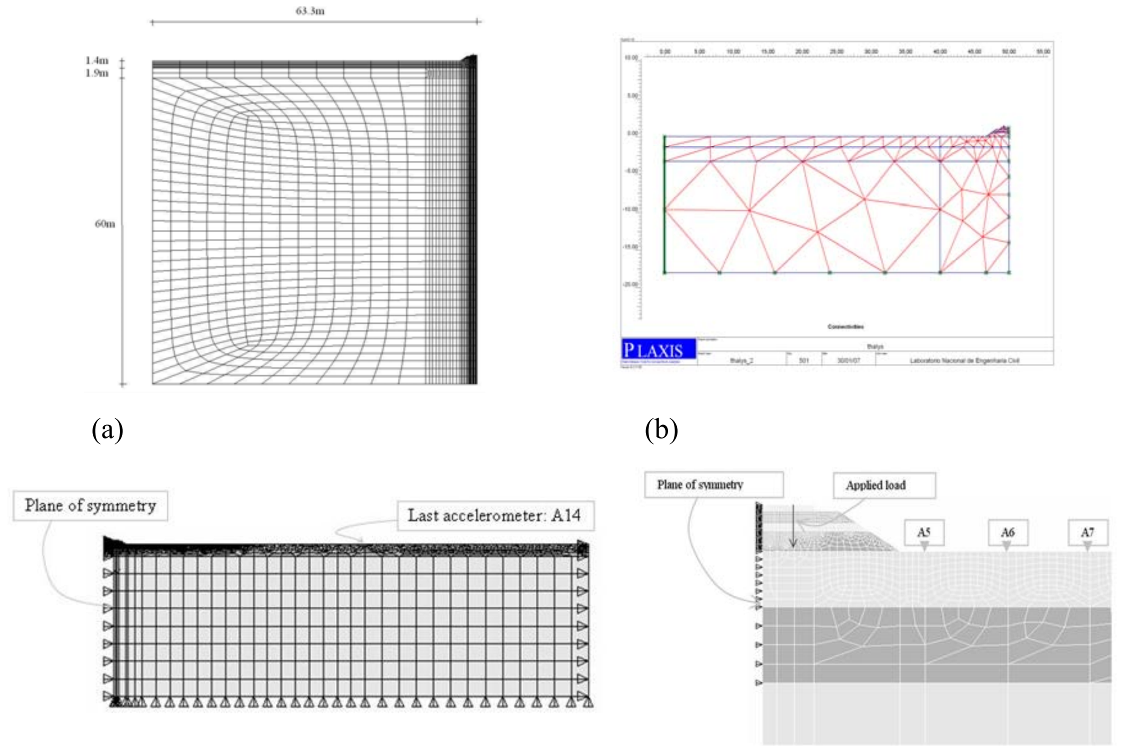
The model used in Plaxis numerical simulations is represented in Figure 7b.

The model is composed of 135 finite elements of 15 nodes, totaling 1183 nodal points. It represents half of the complete model due to its simmetry. The left boundary is absorbent in order to dissipate the incoming vibrations. The right boundary corresponds to the symmetry axis and therefore has null horizontal displacements. The bottom boundary was considered as fixed in order to avoid an overall movement of the model.

Plaxis, as DIANA, considers Rayleigh damping. The same procedure as for DIANA was done to obtain α and β parameters using the same ξ . Then, in a separate modal analysis the first two frequencies obtained using Plaxis were 4.26 and 5.04 Hz, which lead to the ξ, α and β values presented in Table 2. These differences obtained by two different FEM codes using different simplifications in geometry and boundary conditions are under investigation.

The UNL team uses ANSYS code. For the model structural plane, elements of 4 nodes in plane strain condition were selected, with preferential quadrilateral shape used to avoid additional rigidity inherent to triangle shapes. Finite element mesh is shown in Figs. 7c and 7d; in all, 3210 elements with side ranging from 0.05m to 5.00m were generated.

Non-reflecting boundary conditions are not available in the code. For that reason, the thickness of the last layer and the soil beyond the last accelerometer is represented to a depth and width of 40m and 64m, respectively. On the “infinite” boundaries normal displacements were restrained. Advantage is taken of symmetry. As for the previous models materials are linear elastic and isotropic. In this code, in full transient analysis, only Rayleigh damping, material dependent damping and element damping are accessible. A first exercise was carried out using material dependent damping taking different values for distinct materials, but that cannot change over the analysis. The results obtained were not realistic. At that stage some preliminary work was done by the UNL team, changing some of the features of the ANSYS model by comparing results for different options. Surface displacements, velocities and accelerations along model width for specific time steps were used for comparison at times 0.140, 0.175, 0.301s, 0.335, 0.373 and 0.407s that correspond to the first local peaks of the applied load. Calculation was stopped at 0.5s, when the load stabilized at zero value.



(a) (b)

(c) (d)

Figure 7.\ Finite element mesh for calculations: a) DIANA, b) Plaxis, c and d) ANSYS

It was verified that the rail, including the interface material, approximated by three springs in the way to preserve the total stiffness 100MN/m per 0,6m, does not bring any changes to the results.

It was also confirmed that the size of the elements reported above in the ANSYS model is sufficient, with a total of 66296 elements leading to unaltered values.

The representation of the boundary conditions influence was also examined changing the thickness and width of represented soil layer (extended to 80m and to 128m beyond last accelerometer). Slight differences in displacement field were detected while acceleration values were practically unchanged.

The next exercise was to adopt a Rayleigh damping which results are presented hereafter. In this case α and β damping constants are introduced in analysis or load step, but they cannot change for different materials. However, they can vary in each time step. The values adopted for all the materials were $\alpha = 0.870339$ and $\beta = 0.001027$.

Full transient analyses were performed with the different FEM codes and results shown in Figures 8, 9 and 10.

The comparison between the predictions can be summarized as presented in Table 3. The peak accelerations estimated by DIANA, Plaxis and ANSYS are rather close. However, the estimated values have rates very different from unity related with the measured values, particularly for the acceleration on the sleeper. Several reasons can contribute to this discrepancy, the major being:

a) simplifying assumptions made (dynamic loading, interface between sleeper and ballast rail – rail pad;
b) differences between material values;
c) differences in the boundary conditions;
d) simplifying dynamic material behaviour.

These factors need further study to better identify those that need modification. Additional studies on the influence of several parameters are currently under development and the possibility of using different computational codes is being considered.

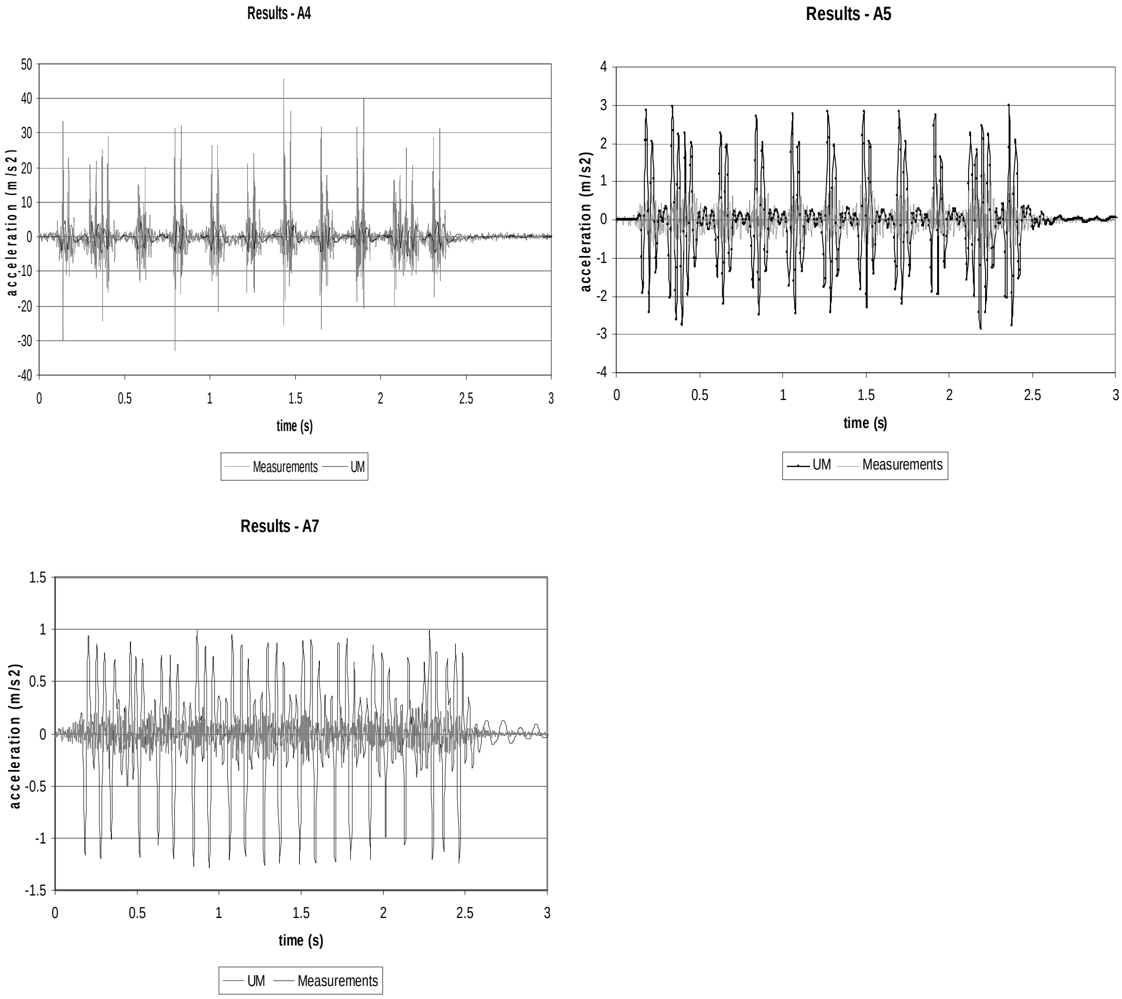
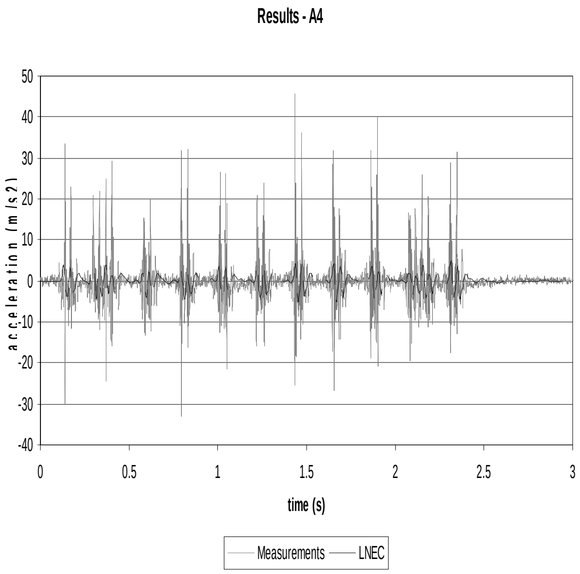
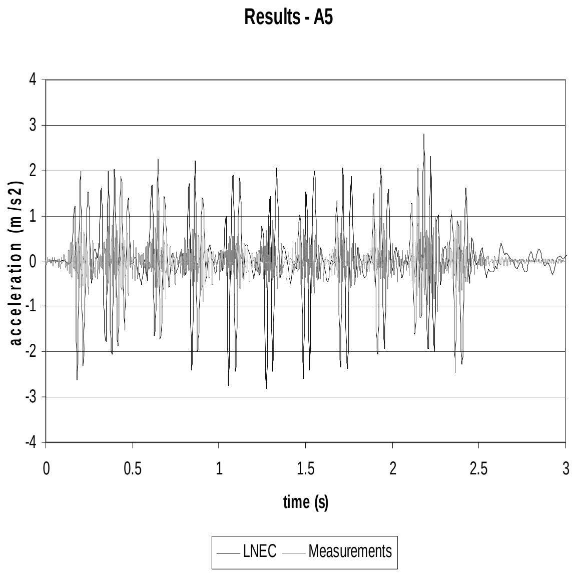


Figure 8. Results by DIANA (UM)





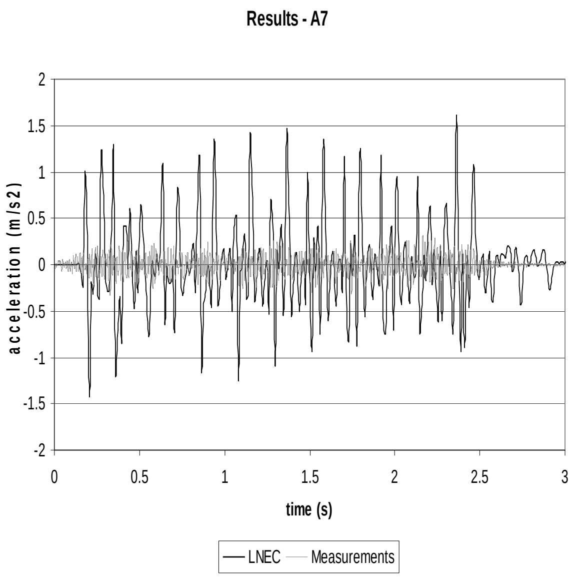


Figure 9. Results by Plaxis (LNEC)

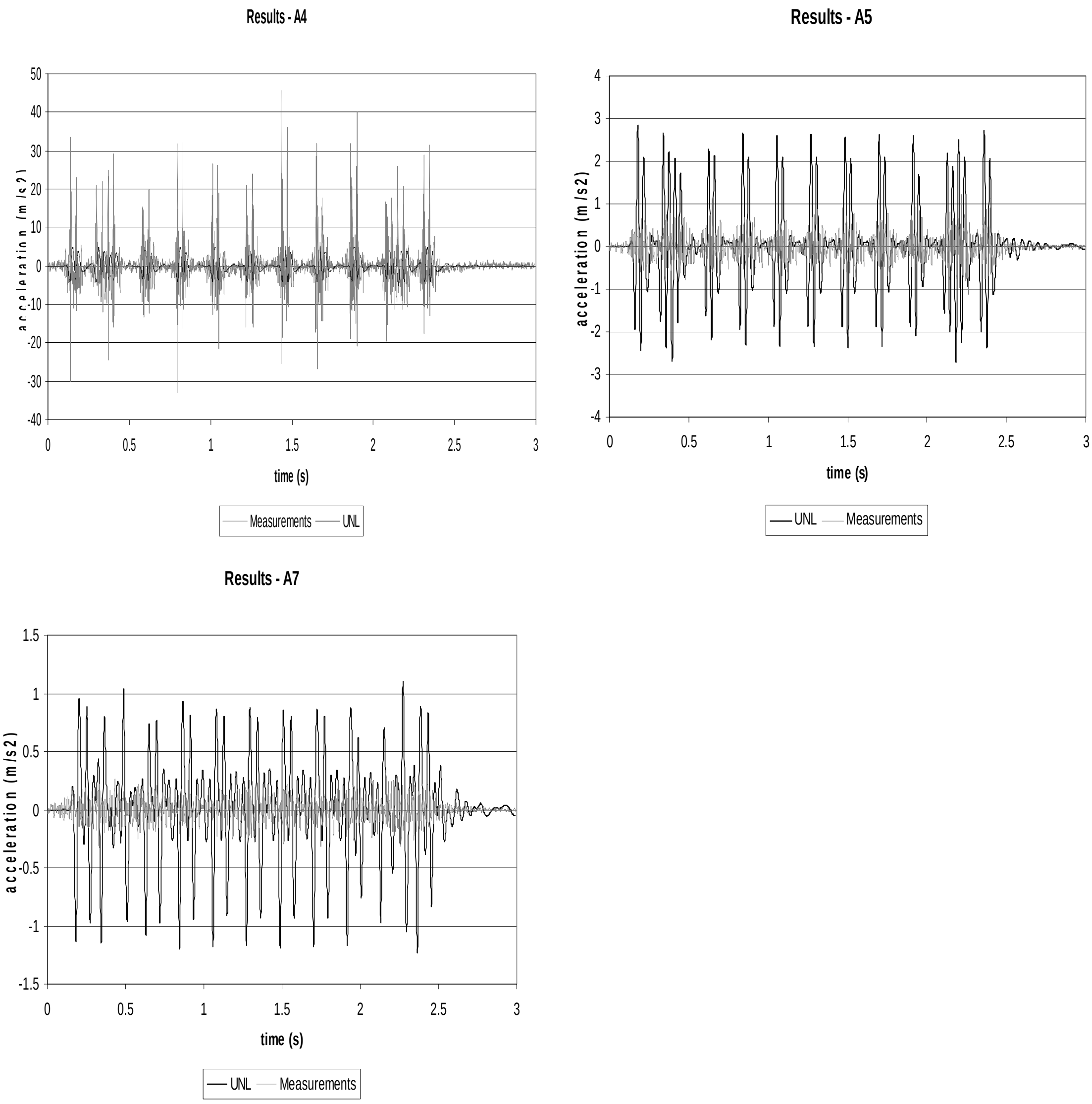


Figure 10. Results by ANSYS (UNL)

3 CONCLUSION

The results are presented here for railtrack modelling using three FEM codes. The general trends in prediction of rail track performance and induced vibrations in the nearly area (soil) generally agree between the three models used but there are also some very significant variations in the relative rates for the measured values, particularly for sleeper response.

The ongoing research work in the project involving cooperation between specialists in soil dynamics and structural dynamics is a guarantee of the further improvements in modelling. Some significant aspects considered for further developments include: (a) simulation of dynamic loading, (b) vibrations characteristics and dynamic interaction of concrete sleepers and ballast support system and (c) frequency and strain dependence of the materials stiffness and damping.

Table 3. Rates between estimations by different FEM codes and estimations and measurements for maximum peak accelerations at different points

	Values m/s²	Rates A4		
		DI-ANA	Plax-is	ANSYS
Measure-ments	45.67	0.1	0.1	0.1
DIANA	4.74	-	1.0	1.0
Plaxis	4.82	-	-	1,0
ANSYS	4.96	-	-	-

	Values m/s²	Rates A5		
		DI-ANA	Plax-is	ANSYS
Measure-ments	1.23	2.4	2.3	2.3
DIANA	2.99	-	0.9	0.9
Plaxis	2.80	-	-	1.0
ANSYS	2.84	-	-	-

	Values m/s²	Rates A7		
		DIANA	Plax is	ANSYS
Measure-ments	0.32	3.1	5.0	3.5
DIANA	0.99	-	1.6	1.1
Plaxis	1.61	-	-	0.7
ANSYS	1.11	-	-	-

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