100

Seasonal Hunting and Food Fluctuations in the Lotka-Volterra Model

Attila Genda[1], Alexander Fidlin[1], Stefano Lenci[2], Oleg Gendelman[3]
[1]Karsruhe Institute of Technology, Karlsruhe, Germany. [2]Polytechnic University of the Marche, Ancona, Italy. [3]Technion - Israel Institute of Technology, Haifa, Israel

Abstract

The Lotka-Volterra model, introduced by Alfred J. Lotka [1] and Vito Volterra [2] in the 1920s, remains a fundamental framework for studying predator-prey interactions. Over the years, researchers such as Ikeda [3] and Redheffer [4] extended the model by incorporating time-dependent coefficients to reflect real-world influences like hunting and food availability.

In this work, we examine the impact of periodic hunting and food supply variations on the Hamiltonian Lotka-Volterra model by introducing a time-dependent prey reproduction rate, which is modeled as a periodically varying coefficient.

$x'=\alpha(t)x-xy$,
$y'=xy-y$,
with $\alpha(t)=\Omega^2+f\cdot\sin(\omega t+\beta)$, where $F=f/\varepsilon$ and $\omega=\Omega(1+\Delta\sqrt{\varepsilon})$. Here, Ω^2 represents the non-dimensional prey per capita growth rate in the unperturbed system, while ω is the excitation frequency near the system's natural frequency.

Our analysis reveals that in specific parameter ranges, significant nonlinear oscillations emerge, leading to critical population thresholds. When these thresholds are crossed, the risk of species extinction increases significantly. We determine the conditions under which this occurs by identifying the critical forcing amplitude F_{crit} as a function of the frequency discrepancy Δ.

Using perturbation techniques and 1:1 resonance analysis, we reduce the system to action-angle variables [5], enabling a more tractable analytical study. The results demonstrate that predator-prey populations can be destabilized within certain frequency ranges of hunting or food supply variations. Our findings align well with numerical simulations and provide practical guidelines for managing biological systems. This can help policymakers define sustainable hunting limits and regulate food supply fluctuations to prevent population collapse.

References:

[1] Lotka, A. J. (1920). *Proceedings of the National Academy of Sciences*, 6(7), 410–415.
[2] Volterra, V. (1926). *Memoria della Reale Accademia Nazionale dei Lincei*, 2, 31–113.
[3] Ikeda, M., & Siljak, D. D. (1979). *IEEE Conference on Decision and Control*, 2, 593–601.
[4] Redheffer, R. (1996). *Journal of Differential Equations*, 127, 519–541.
[5] Gendelman, O. V. (2018). *Nonlinear Dyn*, 93, 79–88.

Categories

TND: Transient and Non-stationary Dynamics