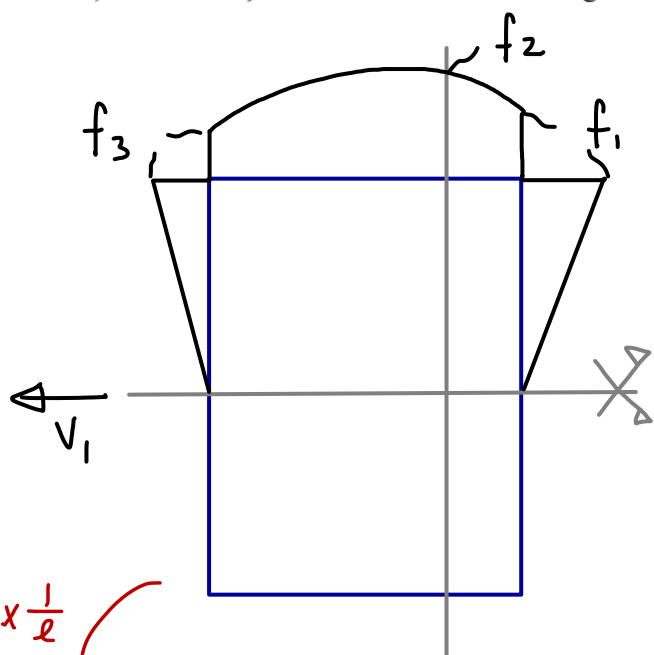


$$I_1 = 2,595 \times 10^{-4} \text{ m}^4$$

$$I_2 = 1,819 \times 10^{-4} \text{ m}^4$$

- a) Represente o diagrama de tensões tangenciais da secção para:
a.1) um esforço transversal de 500 kN segundo o eixo x_1 ;



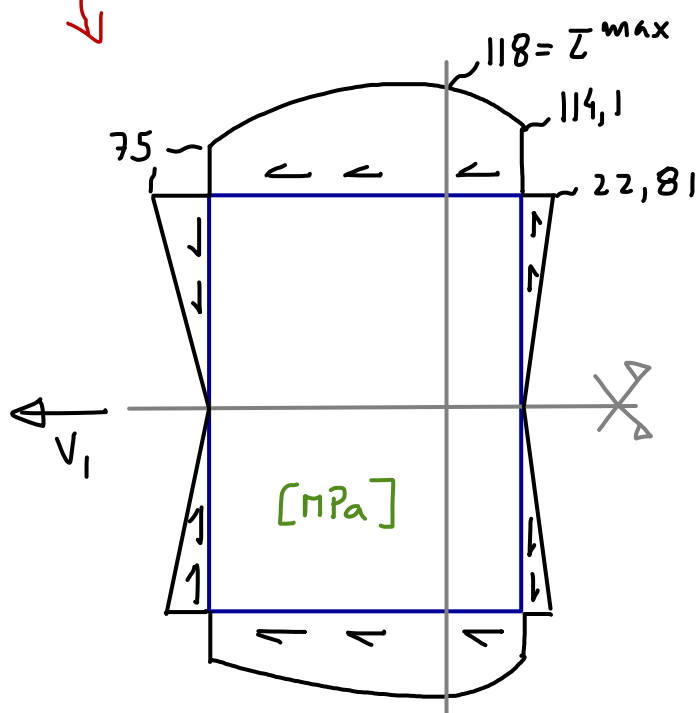
$$f_1 = \frac{V_1 \times 5,353 (15,5 \times 5) \times 10^6}{1,819 \times 10^{-4}} = 2,281 V_1$$

$$f_3 = \frac{V_1 \times 17,647 (15,5 \times 1) \times 10^6}{1,819 \times 10^{-4}} = 1,5 V_1$$

$$f_2 - f_1 = \frac{V_1 \times 5,353^2 \frac{1}{2} \times 1 \times 10^{-6}}{1,819 \times 10^{-4}} = 0,07876 V_1$$

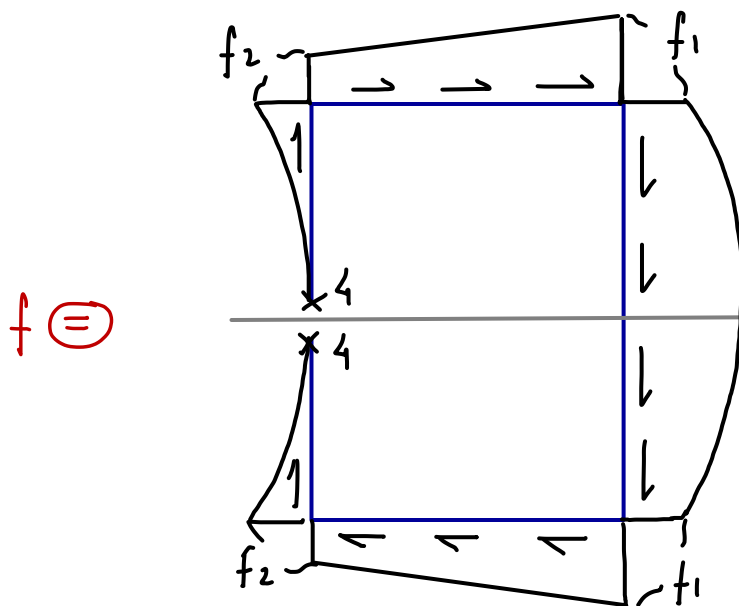
$$\Downarrow$$

$$f_2 = 2,36 V_1$$

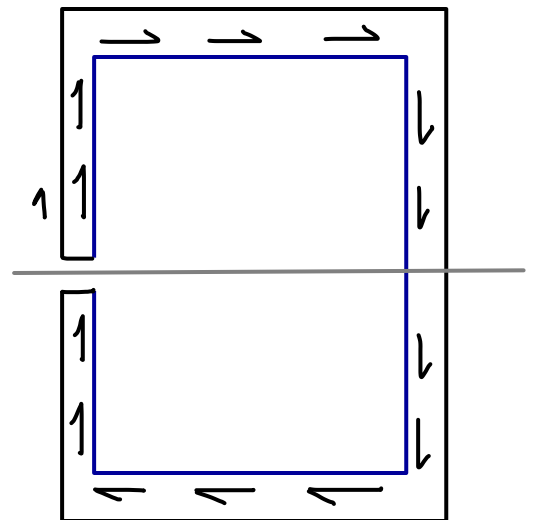


a.2) um esforço transverso de 500 kN segundo o eixo x_2

Solução particular



Solução complementar



$$f_2 = \frac{V_2 \times 15,5^2 \frac{1}{2} \times 1 \times 10^{-6}}{2,595 \times 10^{-4}} = 0,463 V_2$$

$$f_1 - f_2 = \frac{V_2 \times 23 \times 1 \times 15,5}{2,595} \times 10^{-2} \Rightarrow f_1 = 1,857 V_2$$

$$f_3 - f_1 = \frac{V_2 \times 15,5^2 \frac{1}{2} \times 5}{2,595} \times 10^{-2} \Rightarrow f_3 = 4,172 V_2$$

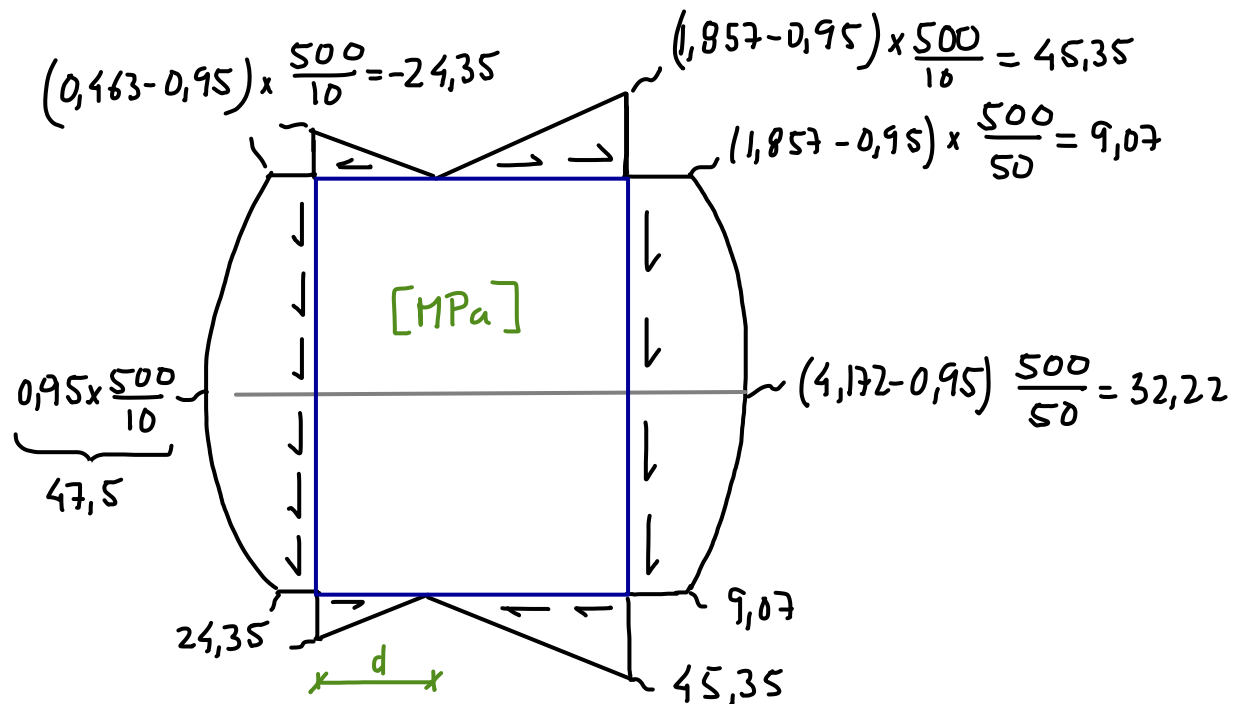
$$\Delta u = 0 \Rightarrow \Delta u_p + f_4 \times \Delta u_c = \frac{1}{G} \oint \frac{f_p}{e} ds + \frac{f_4}{G} \oint \frac{f_c}{e} ds \Leftarrow \text{eq. compatibilidade de}$$

$$\begin{aligned} \frac{1}{e} \int \frac{f_2}{15,5} ds &+ \frac{1}{e} \int \frac{f_1}{23} ds + \frac{1}{e} \int \left(\frac{f_1}{31} + \frac{f_3 - f_1}{31} \right) ds \\ G \cdot \Delta u_p &= 2 \times \frac{0,463 V_2}{0,01} \times \frac{1}{3} \times 0,155 + 2 \times \frac{\frac{1}{2} (1,857 + 0,463) V_2 \times 0,23}{0,01} + \frac{V_2}{0,05} \times 0,31 \left[1,857 + \frac{2}{3} (4,172 - 1,857) \right] \\ &= V_2 (4,63 + 53,36 + 21,08) = 79,07 V_2 \end{aligned}$$

$$G \cdot \Delta u_c = \frac{0,31}{0,01} + 2 \times \frac{0,23}{0,01} + \frac{0,31}{0,05} = 83,2$$

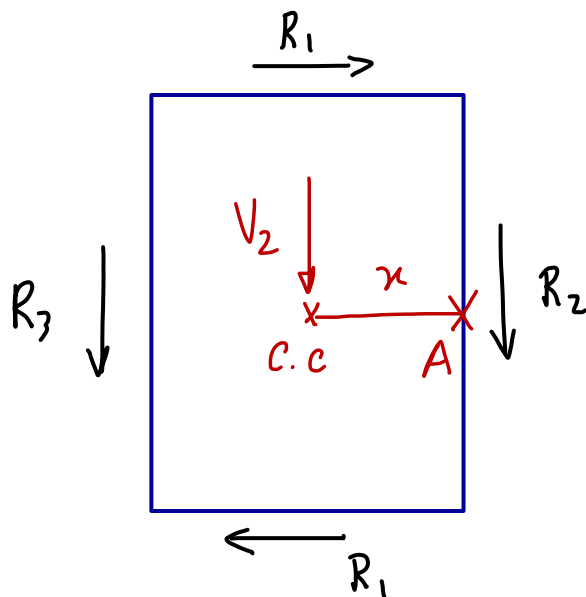
$$\Delta u_p + f_4 \times \Delta u_c = 0 \Rightarrow f_4 = -\frac{79,07 V_2}{83,2} = -0,95 V_2$$

V_2



$$\left. \begin{array}{l} d = 45,35 \\ 23 = 24,35 + 45,35 \end{array} \right\} \Rightarrow d = 8,035 \text{ cm}$$

b) Determine a posição do centro de corte da secção.



$$R_1 = e \int \frac{45,35}{23-d} - \frac{24,35}{d} dA$$

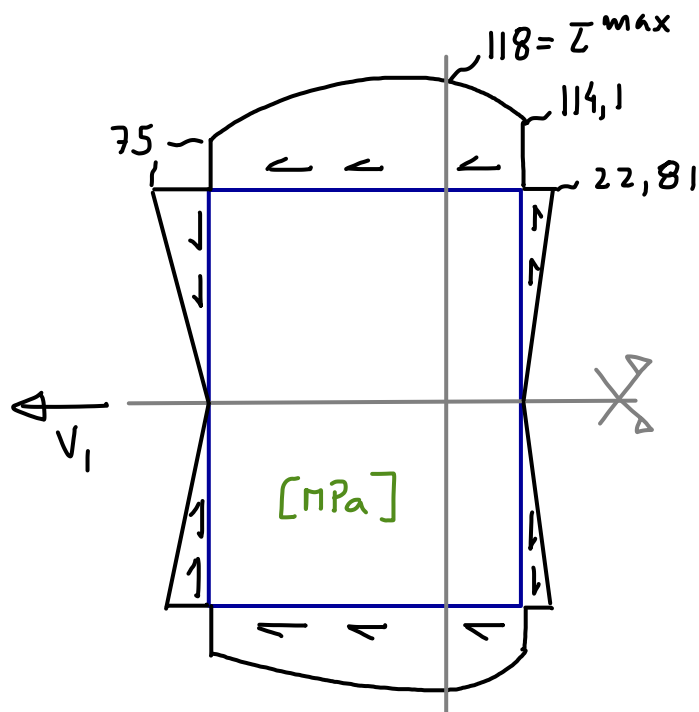
$$R_1 = 10 \times \frac{1}{2} [-24,35d + 45,35(23-d)] \times 10/1000 = 24,15 \text{ kN}$$

$$R_3 = e \int \frac{24,35}{31} + \frac{47,5-24,35}{31} dA$$

$$R_3 = 10 \times 310 \left[24,35 + \frac{2}{3} (47,5 - 24,35) \right] / 1000 = 123,3 \text{ kN}$$

$$\sum M_A \rightarrow 123 \times 23 - 24,15 \times 31 = x \times 500 \Rightarrow x = 4,161 \text{ cm}$$

c) Calcule a área de corte segundo a direcção 1.



$$A_{v1}^* = \frac{V_1^2}{\int_{\Omega} \tau_1^2 d\Omega}$$

$$\begin{aligned} \int_{\Omega} \tau_1^2 d\Omega &= 2 \times \int \left(\frac{75}{15.5} \right)^2 + 5 \times \left(\frac{22.81}{15.5} \right)^2 + \left(\frac{75}{17.647} + \frac{118-75=43}{15} \right)^2 + \left(\frac{114.1}{5.353} + \frac{118-114.1=3.9}{15} \right)^2 d\Omega \\ &= 2 \left[\frac{1}{3} 75^2 \times 15.5 + \frac{5}{3} 22.81^2 \times 15.5 + 17.647 \times \left(75^2 + 2 \times \frac{2}{3} 75 \times 43 + \frac{8}{15} 43^2 \right) + \right. \\ &\quad \left. 5.353 \times \left(114.1^2 + 2 \times \frac{2}{3} 114.1 \times 3.9 + \frac{8}{15} 3.9^2 \right) \right] \\ &= 61833 \text{ kN}^2/\text{cm}^2 \end{aligned}$$

$$A_{v1}^* = \frac{500^2}{6183.3} = 40.43 \text{ cm}^2$$