

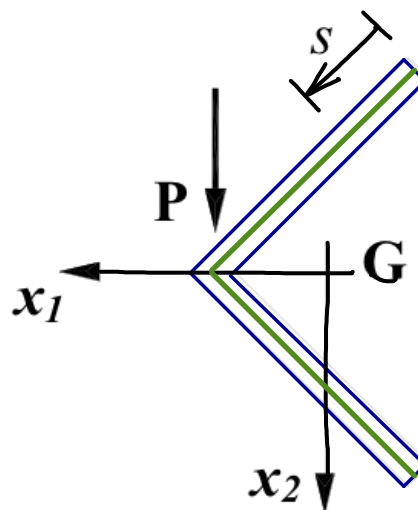
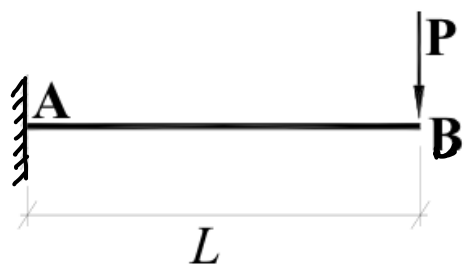
$$L=2m$$

$$P=500 \text{ kN}$$

$$E=210 \text{ GPa}$$

$$\nu=0,3$$

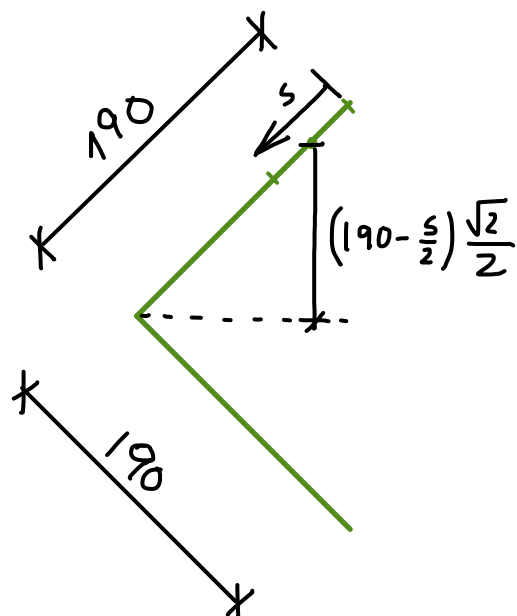
$$G = \frac{E}{2(1+\nu)} \approx 80,77$$



$$200 \times 200 \times 20$$

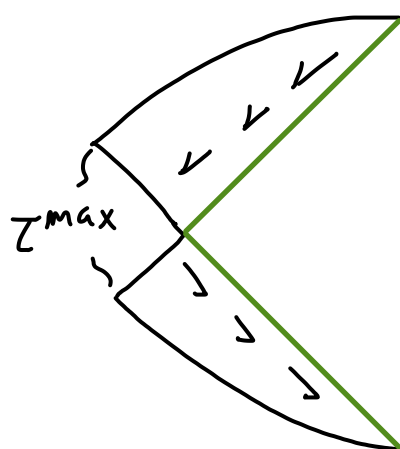
$$I_1 = 45,32 \times 10^6 \text{ mm}^4$$

$$\tau = \frac{V_2 S_1}{I_1 l}$$



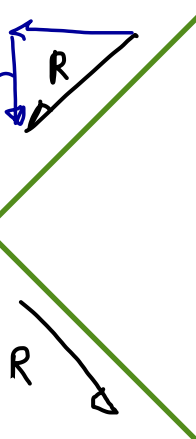
$$\tau = \frac{500 \times 10^3}{45,32 \times 10^6} \times \frac{(20 \times s) \left(190 - \frac{s}{2}\right) \frac{\sqrt{2}}{2}}{20}$$

$$\tau^{\max} = \tau(s=190) \approx 139,2 \text{ MPa}$$



$$\frac{500}{2} = R \frac{\sqrt{2}}{2}$$

CENTRO  
DE  
CORTE (\*)



$$R = \int_0^{190} \tau \times l \, ds = \frac{500}{\sqrt{2}} = 353,6 \text{ kN}$$

$$\int \frac{139,2}{190} \times 1 \, ds$$

(\*) Regras (para secção de parede fina): se (i) todos os troços convergirem num ponto ou (ii) existir plano de simetria, então o centro de corte localiza-se obrigatoriamente nessas zonas

## ÁREA DE CORTE:

$$U = \frac{1}{2} \int_V \varepsilon \cdot \sigma \, dV + \frac{1}{2} \int_V \gamma \cdot \tau \, dV = \frac{1}{2} \int_L \int \frac{\sigma^2}{E} \, d\Omega \, dx_3 + \frac{1}{2} \int_L \int \frac{\tau^2}{G} \, d\Omega \, dx_3$$

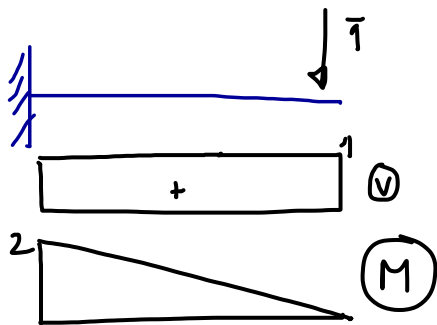
$$\Rightarrow \underbrace{\frac{1}{2} \int_L \int \frac{N^2}{EA^2} \, d\Omega \, dx_3}_{\int_L \frac{N^2}{EA} \, dx_3} + \underbrace{\frac{1}{2} \int_L \int \frac{M^2}{EI^2} \, d\Omega \, dx_3}_{\int_L \frac{M^2}{EI} \, dx_3} + \underbrace{\frac{1}{2} \int_L \int \frac{V^2}{G} \frac{S^2}{I^2 e^2} \, d\Omega \, dx_3}_{\int_L \frac{V^2}{G A^*} \, dx_3}$$

$$A^* = \frac{I^2}{\int \frac{S^2}{e^2} \, d\Omega} = \frac{I^2}{\int \frac{S^2}{e} \, d\Omega} = \underline{\underline{\frac{V_z^2}{\int \tau^2 \, d\Omega}}}$$

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$$\int_{\Omega} \tau^2 \, d\Omega = 2e \int \left( \frac{139,2}{190} \right)^2 \, dx_3 = 0,02 \times 2 \times \frac{8}{15} (139,2 \times 10^3)^2 \times 0,19 = 78,54 \times 10^6 \, [\text{kN}^2/\text{m}^2]$$

$$A^* = \frac{500^2}{78,54 \times 10^6} = 3,183 \times 10^{-3} \, \text{m}^2$$



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$$\delta = \int_L \frac{M \bar{M}}{EI} + \frac{V \bar{V}}{GA^*} \, dx_3$$

$$\delta = \int \frac{1}{EI} \left[ \frac{2 \times 500}{2} \right]^2 \, dx_3 + \frac{1}{GA^*} \left[ \frac{500}{2} \right]^2 \, dx_3$$

$$\delta = \underbrace{\frac{1}{EI} \frac{1}{3} 8 \times 500}_{140,1 \, \text{mm}} + \underbrace{\frac{1}{GA^*} 500 \times 2}_{3,89 \, \text{mm}} = 143,99 \, \text{mm}$$

$$\frac{143,99 - 140,1}{143,99} \times 100 = 2,7 \%$$