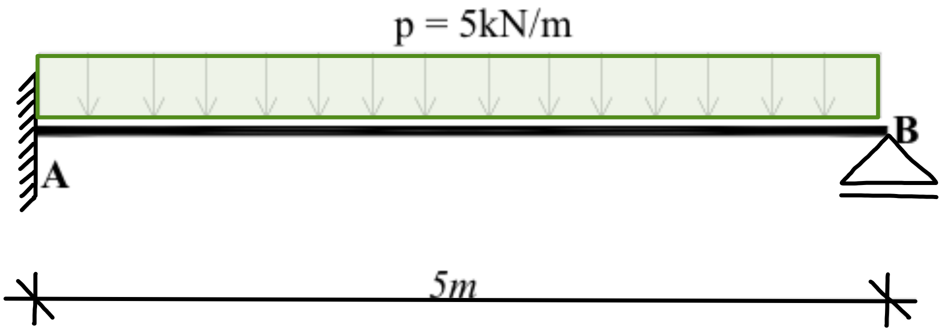
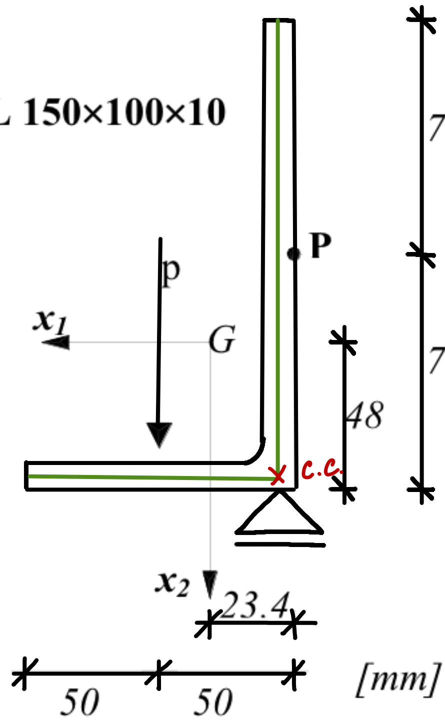




$I_1=5,52x10^6\ mm^4$
$I_2=1,98x10^6\ mm^4$
$I_{12}=-1,92x10^6\ mm^4$

$$V_B=\frac{3}{8}pL$$

$$V_A=\frac{5}{8}pL=15,63$$

$$M_A=-5\times5\times2,5+5\times V_B=-15,63\ kNm$$

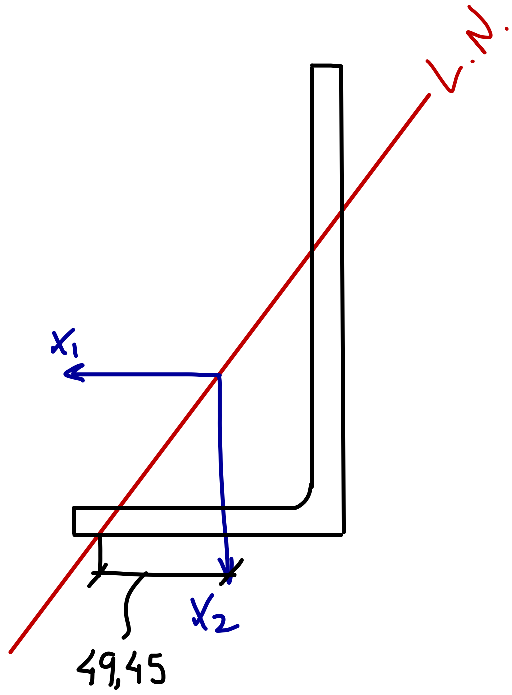
$$I_{11}I_{22}-I_{12}^2=7,243\times10^{12}\ mm^8$$

$$T_{A,cc}=5\times5\times0,045=1,125\ kN.m$$

$$\boxed{\sigma=\frac{M_1I_{12}-M_2I_{11}}{I_{11}I_{22}-I_{12}^2}x_1+\frac{M_1I_{22}-M_2I_{12}}{I_{11}I_{22}-I_{12}^2}x_2}$$

$$\sigma=M_1\left(\frac{-1,92}{7,243}x_1+\frac{1,98}{7,243}x_2\right)=M_1\left(-0,265x_1+0,273x_2\right)$$
[MPa]

$$\sigma_{A,P}=-15,63\times\left[-0,265\times(-23,4)+0,273\times(-75+48)\right]=+18,41MPa$$

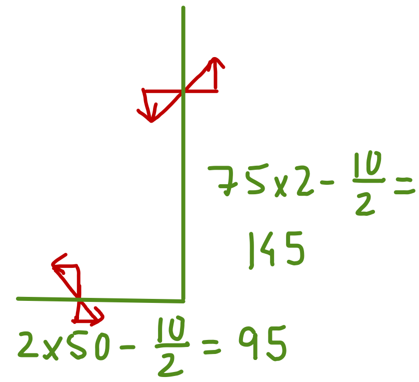


$$0=-0,265x_1+0,273x_2\Rightarrow x_2=0,971x_1$$

$$x_2=48\Rightarrow x_1=\frac{48}{0,971}=49,45$$

b. Determine o valor da tensão de comparação, segundo von Mises, do ponto **P** da secção do apoio **A**.

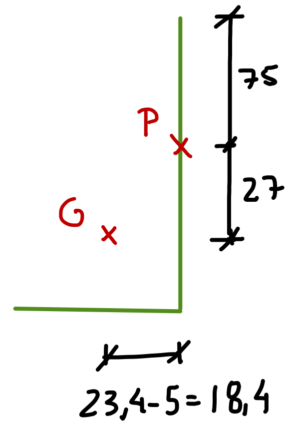
$\sigma_{A,P} = 18{,}41 \text{ MPa}$



$$J = \sum \frac{l t^3}{3} = \frac{1}{3}(95+145)\times 10^3 = 80\times 10^3 \text{ mm}^4$$

$$\tau = \frac{T}{J} e = \frac{1{,}125\times 10^6}{80\times 10^3}\times 10 = 140{,}6 \text{ MPa}$$

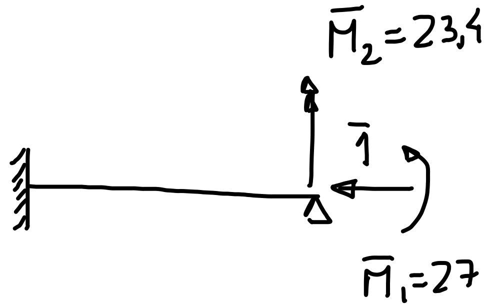
$$\boxed{\tau_V = -V_2 \frac{I_2 S_1 + I_{12} S_2}{I_1 I_2 - I_{12}^2} - V_1 \frac{I_1 S_2 + I_{12} S_1}{I_1 I_2 - I_{12}^2}}$$

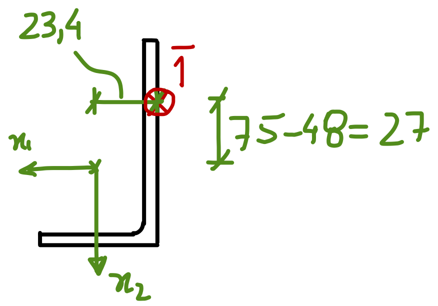


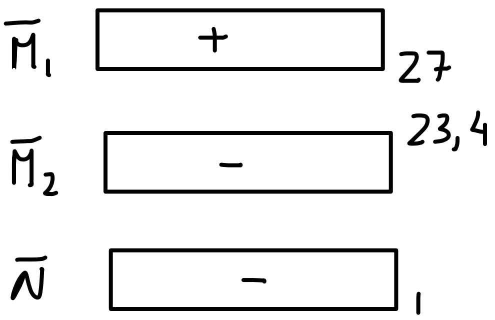
$$\tau = -15{,}63\times 10^3 \times \frac{1{,}98\times 10^6 \times 75 \times (-75/2 - 27) + (1{,}92\times 10^6)\times 75\times(-18{,}4)}{7{,}243\times 10^{12}} = 14{,}95 \text{ MPa}$$

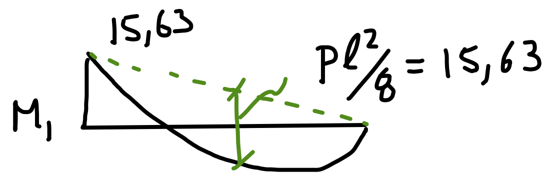
$$\sigma_{vm} = \sqrt{\sigma^2 + 3\tau^2} = \sqrt{18{,}41^2 + 3(14{,}95 - 140{,}6)^2} = 218{,}4 \text{ MPa}$$

c. Calcule o deslocamento horizontal do ponto **P** situado na secção do apoio **B**.









$$\boxed{\sigma = \frac{M_1 I_{12} - M_2 I_{11}}{I_{11} I_{22} - I_{12}^2}\, x_1 + \frac{M_1 I_{22} - M_2 I_{12}}{\underbrace{I_{11} I_{22} - I_{12}^2}_{\beta}}\, x_2}$$

$$PTV \Rightarrow \delta_P = \int_V \varepsilon\, \overline{\sigma}\, dV \Rightarrow \frac{1}{E}\int \sigma\, \overline{\sigma}\, dV =$$

$$= \frac{1}{E}\int_V M_1 \frac{(I_{12} x_1 + I_{22} x_2)}{\beta} \cdot \left[\overline{M}_1 \frac{(I_{12} x_1 + I_{22} x_2)}{\beta} - \overline{M}_2 \frac{(I_{11} x_1 + I_{12} x_2)}{\beta} + \frac{\overline{N}}{A}\right] dV$$

$\int_V dV = \int_L \int_A dA\, dx_3$; NOTA: $\int_A x_1\, dA = \int_A x_2\, dA = 0$; $\int_A x_1^2\, dA = I_{22}$; $\int_A x_2^2\, dA = I_{11}$; $\int_A x_1 x_2\, dA = -I_{12}$

$$= \frac{1}{E}\int_L M_1 \overline{M}_1 \underbrace{\frac{(I_{22}^2 I_{11} - I_{12}^2 I_{22})}{(I_{11} I_{22} - I_{12}^2)^2}}_{9{,}613\times 10^{-5}} + M_1 \overline{M}_2 \underbrace{\frac{(-I_{12}^3 + I_{22} I_{11} I_{12})}{(I_{11} I_{22} - I_{12}^2)^2}}_{8{,}0791\times 10^{-5}}\, dx_3$$

$$= \frac{1}{E}\; 5\left(-\frac{1}{2} + \frac{2}{3}\right) 15{,}63\times 10^6 \left[27 \times 9{,}613\times 10^{-5} - 23{,}4 \times 8{,}0791\times 10^{-5}\right]$$

$$= \frac{176{,}92}{E}\ mm\ (\leftarrow)$$

MPa