A Metaheuristic for Solving Large-Scale Two-Stage Stochastic Mixed 0-1 Programs with a Time Consistent Stochastic Dominance Constraints Risk Averse Strategy

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Abstract

In the present paper, we study general two-stage stochastic mixed 0-1 problems, in the context of closed loop systems where forward and reserve chains are simultaneously considered. Besides the fact that uncertainty appears anywhere in the objective function, right hand side and constraint matrix coefficients, the 0-1 variables and continuous variables have nonzero coefficients in both stages. A metaheuristic algorithm is proposed as a specialization for two-stage problems of the so named Fix-and-Relax Coordination Algorithm (FRCA) for solving large-scale multiperiod stochastic mixed 0-1 optimization problems under a time stochastic dominance constraint strategy, so-named TSD. This strategy is a mixture of the first and second-order stochastic dominance constraint risks averse measures induced by mixed-integer linear recourse that are considered in [Gollmer et al., 2011] plus enduring this strategy along the time horizon see [L.F. Escudero and Pérez, 2014]. In order to assess the solution procedure applicability, computational tests were performed on multi-period and multi-commodity closed loop supply chains randomly generated from a deterministic reference case previously addressed in [Salema et al., 2010]. Due to the large scale of the deterministic case involved, the stochastic counterparts addressed are very large ones.

Keywords: Two-stage stochastic mixed 0-1 optimization, metaheuristic, Fix and Relax, time stochastic dominance

1. Introduction

Supply Chains are complex networks of entities and flows that recently have evolved to consider closed loop systems where forward and reserve chains are simultaneously considered. Under this extended systems the treatment of uncertainty has been recognized as crucial although the resulting problems, usually large-scale two-stage stochastic problem result in complex system to be solved.

In the present paper we aim to contribute to the solution of this problem. We study general two-stage stochastic mixed 0-1 problems, where the uncertainty appears anywhere in the objective function, right and side and constraint matrix coefficients. The 0-1 variables and continuous variables have nonzero coefficients in both stages. A metaheuristic algorithm is proposed as a specialization for two-stage problems of the so named Fix-and-Relax Coordination Algorithm (FRCA)
for solving large-scale multiperiod stochastic mixed 0-1 optimization problems. So, in the multi-
period multistage case as in the multiperiod two-stage case the large-scale character can be moti-
vated by the intrinsic dimensions of the problem as well as due to the large number of additional
0-1 and continuous variables and constraints required by the risk averse strategy to use. In this
work it is the time consistent stochastic dominance constraint strategy, so-named SDC, as mixture
of the first and second-order stochastic dominance constraint risks averse measures induced by
mixed-integer linear recourse that are considered in [Gollmer et al., 2011] plus considering a time
consistency policy, see [L.F. Escudero and Pérez, 2014].

A main characteristic of the inexact approach that is proposed consists of considering so-named
levels along the time horizon included by disjoint sets of consecutive periods. At each level an
independent partial scenario mixed 0-1 model is solved by fixing the 0-1 variables of ancestor
levels to the value obtained at the optimization of their related models and relaxing the integrality
of the 0-1 variables related to successor levels as well as the (cross scenario SDC systems) for
those other levels. Additionally, the continuous variables are obtained at each level by considering
the bound targets of the SDC systems up to that level.

In order to assess the solution procedure applicability, computational tests were performed on
multi-period and multi-commodity closed loop supply chains randomly generated from a deter-
minalistic reference case previously addressed in [Salema et al., 2010], that being a large scale case,
itstochastic counterpart is a very large one.

The rest of the work is organized as follows. In Section 2 the two-stage mixed 0-1 optimization
problem with the risk averse strategy TSD is presented, as well as the model for the risk neu-
tral one. Section 3 presents our specialization of the the Fix-and-Relax Coordination (FRCA) to
the two-stage problem with strategy TSD. Section 4 reports the main results of a computational
experience for assessing the validity of the proposal made in this work. Section 5 concludes.

2. Two-stage stochastic mixed 0-1 problems under time stochastic dominance
(TSD) strategy

It is well known that the risk neutral (expected) objective function maximization has the inconve-
nience of providing a solution that ignores the variability of the function value over the scenarios,
if any. So, it does not hedge against the occurrence of low-probability high-bad consequence
outlooks (i.e., the so named “black swan” events).

Alternatively, there are some other approaches that also deal with risk management by providing
hedging solutions against the occurrence of some non-desired scenarios considering risk averse
strategies, see a survey in [Alonso-Ayuso et al., 2014]. Some of these strategies consider semi-
deviations, excess probabilities, conditional value-at-risk, expect shortfall on reaching given thresh-
olds as risk measure-based functions to optimize as well as the two recent new risk averse mea-
sures, namely, the first- and second-order Stochastic Dominance Constraint strategies. Those later
ones are defined by a set of profiles included by thresholds on given profit values and some types
of shortfall related bounds on reaching them.

In this paper we consider the specialization of a time stochastic dominance risk averse strategy
for solving multistage problems introduced in [L.F. Escudero and Pérez, 2014] to the two-stage
environment. That strategy so-named TSD is a time-based mixture of the FSD and SSD strategies.
One of the main differences of the new strategy and the other ones are the bound on the probability
of failure of reaching the thresholds, the expected shortfall on reaching them and the maximum
shortfall are not hard constrains but targets to reach.

Without loss of generality, let the following compact representation of the two-stage stochastic
mixed 0-1 DEM for maximizing the expected objective function value over the set of scenarios
along a time horizon by only considering the risk neutral (RN) strategy,
\[ z_{\text{RN}} = \max \{ a_1 x_1 + b_1 y_1 + \sum_{\omega \in \Omega} w^{\omega} \sum_{t \in \mathcal{T}^-} (a^{\omega}_t x^{\omega}_t + b^{\omega}_t y^{\omega}_t) \} \]

\text{s.t.} \quad \begin{align*}
A_1^{t\omega} x_1 + B_1^{t\omega} y_1 + \sum_{t' \in \mathcal{T}^- : |t| < t'} (A^{t\omega}_{t'} x^{t\omega}_{t'} + B^{t\omega}_{t'} y^{t\omega}_{t'}) = h^{\omega}_t \quad &\forall t \in \mathcal{T}^-, \omega \in \Omega \quad (1) \\
\omega \in \Omega &\quad (2) \\
x_1 \in \{0,1\}^{n_{\text{ax}(1)}}, y_1 \in \mathbb{R}^{n_{\text{ay}(1)}} &\quad (3) \\
A_1^{t\omega} x^{\omega}_t + B_1^{t\omega} y^{\omega}_t \geq \phi^p &\quad \forall p \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega \\
\sum_{\omega \in \Omega} w^{\omega} \omega^p \leq \nu^p + \epsilon_{\nu}^p, \epsilon_{\nu}^p \in \mathbb{R}_+ &\quad \forall p \in \mathcal{P}, \omega \in \Omega \\
\sum_{\omega \in \Omega} w^{\omega} \omega^p \leq \nu^p + \epsilon_{\nu}^p, \epsilon_{\nu}^p \in \mathbb{R}_+ &\quad \forall p \in \mathcal{P}, t \in \mathcal{T}. 
\end{align*} \]

where \( w^{\omega} \) is the modeler-driven weight or likelihood of scenario \( \omega \); \( x_1 \) and \( y_1 \) are the vectors of the first stage 0-1 and continuous variables, respectively; \( a_1 \) and \( b_1 \) are the objective function coefficient vectors for the variables in the first stage vectors \( x_1 \) and \( y_1 \), respectively; \( A_1^{t\omega} \) and \( B_1^{t\omega} \) are the first stage constraint matrices for the variables in the vectors \( x_1 \) and \( y_1 \), respectively; \( x^{\omega}_t \) and \( y^{\omega}_t \) are the vectors of the second stage 0-1 and continuous variables, respectively; \( A^{t\omega}_t \) and \( B^{t\omega}_t \) are the objective function coefficients for the variables in the second stage vectors \( x^{\omega}_t \) and \( y^{\omega}_t \), respectively; \( A_1^{t\omega} \) and \( B_1^{t\omega} \) are the second stage constraint matrices for the variables in the vectors \( x_1 \) and \( y_1 \); \( A^{t\omega}_t \) and \( B^{t\omega}_t \) are the second stage constraint matrices for the variables in the vectors \( x^{\omega}_t \) and \( y^{\omega}_t \), respectively, for \( 1 < t' \leq t, t \in \mathcal{T}^-, \omega \in \Omega \); \( h^{\omega}_t \) is the rhs; \( n_{\text{ax}(1)} \) and \( n_{\text{ay}(1)} \) are the number of the first stage 0-1 and continuous variables, respectively; \( n_{\text{x}(t)} \) and \( n_{\text{y}(t)} \) are the number of the second stage 0-1 and continuous variables, respectively, for \( t \in \mathcal{T}^- \).

The model RN (1) aims to maximize the objective function expected value (i.e., mean) alone. The main criticism that can be made to this very popular strategy, as stated above, is that it ignores the variability of the objective function value over the scenarios for the RN solution and, in particular in our case, the "left" tail of the non-wanted scenarios. Strategy TSD requires a set of profiles \{\( p \)\}, say \( \mathcal{P} \), \( \omega \in \mathcal{T} \), given by the tuple \( (\phi^p, S^p, S^p, \nu^p) \), where \( \phi^p \) is the objective function threshold to be satisfied up to time period \( t \), \( S^p \) is the maximum target for the shortfall that is allowed on reaching the threshold up to time period \( t \), \( \nu^p \) is the upper bound target of the expected shortfall on reaching the threshold, and \( \nu^p \) is the shortfall probability bound target. The model can be expressed as follows,

\[ z_{\text{TSD}} = \max \{ a_1 x_1 + b_1 y_1 + \sum_{\omega \in \Omega} w^{\omega} \sum_{t \in \mathcal{T}^-} (a^{\omega}_t x^{\omega}_t + b^{\omega}_t y^{\omega}_t) - \sum_{t \in \mathcal{T}^-} \sum_{p \in \mathcal{P}} (M^{\omega}_p \epsilon_{S}^p + M^{\omega}_p \epsilon_{\nu}^p + M^{\omega}_p \epsilon_{\omega}^p) \} \]

\text{s.t.} \quad \begin{align*}
A_1^{t\omega} x_1 + B_1^{t\omega} y_1 + \sum_{t' \in \mathcal{T}^- : |t| < t'} (A^{t\omega}_{t'} x^{t\omega}_{t'} + B^{t\omega}_{t'} y^{t\omega}_{t'}) = h^{\omega}_t \quad &\forall t \in \mathcal{T}^-, \omega \in \Omega \quad (2) \\
\omega \in \Omega &\quad (3) \\
x_1 \in \{0,1\}^{n_{\text{ax}(1)}}, y_1 \in \mathbb{R}^{n_{\text{ay}(1)}} &\quad (4) \\
A_1^{t\omega} x^{\omega}_t + B_1^{t\omega} y^{\omega}_t \geq \phi^p &\quad \forall p \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega \\
\sum_{\omega \in \Omega} w^{\omega} \omega^p \leq \nu^p + \epsilon_{\nu}^p, \epsilon_{\nu}^p \in \mathbb{R}_+ &\quad \forall p \in \mathcal{P}, t \in \mathcal{T} \\
\sum_{\omega \in \Omega} w^{\omega} \omega^p \leq \nu^p + \epsilon_{\nu}^p, \epsilon_{\nu}^p \in \mathbb{R}_+ &\quad \forall p \in \mathcal{P}, t \in \mathcal{T}, \omega \in \Omega. 
\end{align*} \]

where \( \omega^p \) is the shortfall (continuous) variable that, obviously, is equal to the difference (if it is positive) between threshold \( \phi^p \) and the objective function value up to time period \( t \) for \( p \in \mathcal{P}, t \in \mathcal{T} \) for scenario \( \omega \in \Omega \), \( \nu^p \) is a 0-1 variable such that its value is 1 if the objective function value
up to time period \( t \) is smaller than threshold \( \phi^p \) and otherwise, 0, and \( \varepsilon_p^f \), \( \varepsilon_p^p \) and \( \varepsilon_p^n \) are the non-negative slack variables that take the violations of the \( S^p, S^f \) and \( S^n \)-bounds, respectively, being \( M_p^S, M_p^p \) and \( M_p^n \) the related big enough \( M \)-parameters for the penalization of that slack variables in the objective function. Observe that the time stochastic dominance strategy-based model object consists of controlling the objective function value at modeler-driven periods, instead of only performing it at the end of the time horizon. It is very useful for applications with long horizons.

Notice that the mixture of the FSD and SSD related constraints avoids the drawback of not having control on the expected shortfall over the scenarios up to given time periods in case of considering solely FSD. It also avoids the drawback of not having control on the fraction of scenarios with shortfall on reaching the thresholds in case of considering solely SSD.

3. TSD-FRCA2: a Fix and Relax Coordination algorithm for two-stage stochastic mixed 0-1 problems under TSD strategy

The main characteristic of the inexact approach that is proposed consists of considering so-named levels along the time horizon included by disjoint sets of consecutive periods. At each level an independent partial scenario mixed 0-1 model is solved by fixing the 0-1 variables of ancestor levels to the value obtained at the optimization of their related models and relaxing the integrality of the 0-1 variables related to successor levels as well as the (cross scenario SDC systems) for those other levels. Additionally, the continuous variables are obtained at each level by considering the bound targets of the SDC systems up to that level. More formally, the algorithm is formally described by:

**Step 0:** (Initialization). Parameter \( m_{back} \).

**Step 1:** (Upper bound \( \bar{z} \) of solution value \( z_{TSD} \)).

- Solve independently all the \( |\Omega| \) scenario submodels (??).
- If any of those submodels is infeasible then the original problem (2) is infeasible as well, STOP.
- Set \( z_{WS} := \sum_{\omega \in \Omega} w_{\omega} z_{\omega} \) as expression (??) and upper bound \( \bar{z} := z_{WS} \).
- Solve the two-stage submodel (??).
- If it is infeasible then the original problem (2) is infeasible as well, STOP.
- Update \( \bar{z} := \min\{z^1, z_{WS}\} \).
- Set \( n_{back} := 0 \) and level \( \ell = 2 \).

**Step 2:** (Forward step)

- If \( \tilde{\mathcal{F}} := \{ f_j : t^f_j \leq t \leq t^e_j \text{ and solve independently all the } |\Omega| \text{ scenario RN models (??);} \}
- else solve model (3) TSD1

**Step 3:** (Detecting infeasibility)

- If any RN model or the modeler-driven TSD1 or TSD2 model is infeasible, then:
- If \( n_{back} = m_{back} \) or \( \ell = 2 \) then STOP: "Original problem status: unknown"; else go to Step 4.
- If \( \ell = |\mathcal{L}| \) then go to Step 5.
- Reset \( \ell := \ell + 1 \).
- Go to Step 2.

**Step 4:** (Backward step)

- Redefine the partition structure: \( t^{f-1}_e := t^f_e, t^{1}_f := t^{i+1}_f, t^{i}_e := t^{i+1}_e \forall i = \ell, \ldots, |\mathcal{L}| - 1, |\mathcal{L}| := \)
|L| - 1.
Reset \( n_{back} := n_{back} + 1 \) and \( \ell := \ell - 1 \).
Go to Step 2.

**Step 3: (Save TSD solution).**

Save \( x_1 = \hat{x}_1, y_1 = \hat{y}_1, \lambda_1 = \hat{\lambda}_1, \gamma_1 = \hat{\gamma}_1, \forall t \in T^-, \omega \in \Omega \).

Save upper bound \( \zeta \).

Compute solution value \( \zeta = a_1 \hat{x}_1 + b_1 \hat{y}_1 + \sum_{\omega \in \Omega} w^{\omega} \sum_{t \in T^-} (a_{1t}^{\omega} x_{1t}^{\omega} + b_{1t}^{\omega} \gamma_t^{\omega}) \).

Compute TSD bounds for all \( p \in \mathcal{P} \), \( t \in \mathcal{T} \); \( \mathcal{T} \) = max_{\omega \in \Omega} \( \mathcal{T}^p \), \( \mathcal{T} \) = \sum_{\omega \in \Omega} w^{\omega} \mathcal{T}^p \) and \( \mathcal{T} \) = \sum_{\omega \in \Omega} w^{\omega} \mathcal{T}^p \) from the solution, and STOP.

where the model (3) is given by:

\[
\zeta_{TSD} = a_1 \hat{x}_1 + b_1 \hat{y}_1 + \max \left\{ \sum_{\omega \in \Omega} w^{\omega} \sum_{t \in T^-} (a_{1t}^{\omega} x_{1t}^{\omega} + b_{1t}^{\omega} \gamma_t^{\omega}) - \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{P}} \left( M^{p} e_{S}^{p} + M^{p} e_{r}^{p} + M^{p} e_{T}^{p} \right) \right\}
\]

s.t. \( A_{1t}^{\omega} x_{1t}^{\omega} + B_{1t}^{\omega} \gamma_t^{\omega} = b_t^{\omega} \quad \forall t \in \mathcal{T}, t \leq t_f, \omega \in \Omega \)

\( x_t^{\omega} = x_1, \gamma_t^{\omega} = \gamma_1 \quad \forall t \in \mathcal{T}^- \)

\( x_t^{\omega} \in \{0, 1\}^{m(t)}, \gamma_t^{\omega} \in \mathbb{R}^{n(t)} \)

\( A_t^{\omega} \in \{0, 1\}^{m(t)} \), \( y_t^{\omega} \in \mathbb{R}^{n(t)} \)

\( a_1 \hat{x}_1 + b_1 \hat{y}_1 + \sum_{t \in \mathcal{T}^-} (a_{1t}^{\omega} x_{1t}^{\omega} + b_{1t}^{\omega} \gamma_t^{\omega}) + s^{\omega} \geq \phi_t^{p} \quad \forall p \in \mathcal{P}, t \in \mathcal{T} : t_f \leq t \leq t_f^e, \omega \in \Omega \)

\( 0 \leq s^{\omega} \leq \mathcal{T} s^{\omega} + e_{\omega}^{p}, s^{\omega} \in \mathbb{R}^{+} \quad \forall p \in \mathcal{P}, \mathcal{T} : t_f \leq t \leq t_f^e, \omega \in \Omega \)

\( \sum_{\omega \in \Omega} w^{\omega} \mathcal{T}^{\omega} \leq \mathcal{T} \hat{e}_{p}^{p} + e_{\omega}^{p}, e_{\omega}^{p} \in \mathbb{R}^{+} \quad \forall p \in \mathcal{P}, \mathcal{T} : t_f \leq t \leq t_f^e, \omega \in \Omega \)

\( \sum_{\omega \in \Omega} w^{\omega} \mathcal{T}^{\omega} \leq \mathcal{T} \hat{e}_{p}^{p} + e_{\omega}^{p}, e_{\omega}^{p} \in \mathbb{R}^{+} \quad \forall p \in \mathcal{P}, \mathcal{T} : t_f \leq t \leq t_f^e, \omega \in \Omega \)

(3)

4. Computational tests

5. Conclusions

6. Acknowledgement

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