A Fix-and-Relax algorithm for solving parallel and sequential versions of a multi-period multi-product closed loop supply chain design and operation planning model

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Abstract In this work we present the sequential and parallel versions of a heuristic algorithm for the solution of a two-stage stochastic mixed 0-1 model for closed loop supply chain planning problem along a time horizon. Some computational experience conducted on randomly generated networks shows the quality of the proposed approach.

Keywords: Closed Loop Supply Chain, Design and Operation Planning, Two-stage Stochastic Mixed 0-1 Optimization

Introduction

The fact that simultaneous design of the forward and reverse channels may lead to significant cost savings, has focused the interest of industry and academia in closed loop supply chains network design. Still, the research in the field has mostly addressed the deterministic case. However, the network parameters are uncertain by nature along a time horizon. Some of the first works addressing the stochastic case are presented in [1] for a single product in a multi-period and in [2] for a single product network in single period. [3] presents a two-stage stochastic model for a multi-product network in single period. The multi-period setting has been recently addressed in [4] and [5]. A recent review on reverse logistics and closed loop supply chain presented in [6] provides a very deep perspective on the work developed in this field. In most published works, a limited number of sources of uncertainty is addressed (mostly, product demand) and the solving procedures are either based on exact commercial solvers that cannot tackle real size problems or on meta-heuristic procedures that are too problem dependent.

In this work we address several sources of uncertainty simultaneously and assume the related random vector to have a discrete distribution, so that the design and planning of a closed loop network is modelled as a two-stage stochastic mixed 0-1 program. Due to the large scale of the problem, a stochastic version of the heuristic Fix and Relax algorithm presented in [7] is introduced in this work to tackle the problem under study. The sequential and parallel versions of the algorithm are also presented. A computational experience is reported in a set of randomly generated networks, where both versions of the algorithm are considered.

Problem description and modelling approach

The modelling framework subject of this paper is an extension of the deterministic closed loop supply chain model introduced in [8] whose risk management is presented in [9]. The forward and reverse supply chains are operated by the same original equipment manufacturer (OEM). Products recovered from clients (end-of-life products) and processed in disassembly centres, are sorted according to their quality: top quality products are sold to a secondary market, good enough products are sent back to plants to be remanufactured and low quality products are disposed. The overall network decisions concern strategic (network topology) and operational issues (network usage planning).

Several sources of uncertainty are taken into account, namely products' demand, returned products' qualities and volumes, transportation costs, available budget for attending the financial costs, annualized amortization of the investment costs in all entities, etc., where entities name plants, distribution and disassembly centres. Since the random vector ξ that pieces together the stochastic components was assumed to have finite support, all its possible realizations are fully described by a set of scenarios, say Ω.

We point out that due to the multi-period (e.g. 15 years) character of the problem and the different nature of the decisions involved, strategic decisions (i.e. location of the three-echelon network and the production capacities) must be specified at pre-defined periods (so named macro-periods) in the time horizon (e.g., years 1, 6 and 11), while all operational decisions (i.e. production and inventory levels and network flows) are to be taken for every period *t* of the time horizon (e.g., a year) given by the set T.

Let the following notation be considered for the two-stage model presented below: a_1 and b_1 denote the objective function coefficients for the 0-1 strategic variables in vector x_1, and the continuous (operation) variables in vector y_1, respectively, a_t^ω and b_t^ω are the related coefficient vectors for the x_t^ω and y_t^ω variables in period t, for scenario $\omega \in \Omega$, w^ω is the weight of scenario ω, A_1 and B_1 are the matrices for the first stage constraint, h_1 is the related rhs, and $A_1^{t\omega}$, $B_1^{t\omega}$, A_t^ω, B_t^ω, h_t^ω are the related elements for the second stage constraint. Then the two-stage model can be expressed by

$$\begin{aligned}
&max \quad a_1 x_1 + b_1 y_1 + \sum_{\omega \in \Omega} w^\omega \sum_{t \in T} a_t^\omega x_t^\omega + b_t^\omega y_t^\omega \\
&\text{s.t.} \quad A_1 x_1 + B_1 y_1 = h_1 \\
&A_1^{t\omega} x_1 + B_1^{t\omega} y_1 + A_t^\omega x_t^\omega + B_t^\omega y_t^\omega = h_t^\omega \qquad \forall t \in T, \omega \in \Omega \\
&x_1 \in \{0,1\}^{nx(1)}, y_1 \in \mathbb{R}^{ny(1)} \\
&x_t^\omega \in \{0,1\}^{nx(t)}, y_t^\omega \in \mathbb{R}^{ny(t)} \;\; \forall t \in T, \omega \in \Omega
\end{aligned} \tag{1}$$

where the objective function is the maximization of the Net Present Value of the expected profit over the scenarios along the time horizon. Notice that the *x* variables represent the strategic decisions of location of all entities that compose the supply chain and the strategic decisions of plants' production capacity that are operated in the network. Since those decisions are to be taken from a discrete set of alternatives, the *x* variables are binary. The *y* variables represent the operational decisions involved, namely the volume of product flows between entities of consecutive levels (from plants to distribution centres, distribution centres to customers, customers to disassemble centres and from this last entity to secondary market, disposal and plants) and the inventory levels at all entities. Finally, the two sets of constraints represent, respectively, the several constraints that have to be ensured in time period 1 and for all periods and scenarios, namely balance equations among entities, upper and conditional lower bound on product flow between entities and on production at plants, stock upper bounding and annualized amortization bounding.

Algorithmic approach

In the algorithm that we propose for solving model (1), a set of levels (denoted as L) is considered along the time horizon defined by disjoint sets of consecutive periods. At each level a mixed 0-1 model (1) is solved by fixing the binary variables of ancestor levels to the value obtained at the optimization of their related models and relaxing the integrality of the binary variables related to successor levels.

Let partition of level set L be such that $L = L_1 \cup L_2$ and $L_1 \cap L_2 = \emptyset$, where L_1 is the set of levels related to first stage and L_2 includes the other levels. We present a rough description of the algorithm where a reductive full backward step is imposed when the full time horizon has been covered. Such backward step reduces the number of entities that exhibit differentiated costs (in the case, plants) and, as pointed in [9], it leads to better results.

Let t^l denote the last period in level l, for $l \in L$ and M_l^ω the model for scenario $\omega \in \Omega$ at level l where ω=0 if $l \in L_1$. Then, for any model M_l^ω the binary variables of ancestor levels are fixed (to the value, say $\hat{x}$) and the integrality of all binary variables of successor levels are relaxed.

Let E denote the set of entities and x_l^e denote the first stage binary variables related to entity e, for $e \in E$, where level l is set to 1. Then, the rough algorithm in its parallel and sequential versions is as follows:

Step 0: Set label *backward :=' false'* and $z := -\infty$.

Step 1: (Solution for first stage levels).

Solve sequentially model M_l^0 $\forall l \in L_1$.

Set parameter *num_e:=* $\sum_{e \in E} \hat{x}_l^e$ (number of opened *e* entities).

Step 2: (Solution for second stage levels).
Parallel version:
Solve in parallel the $|\Omega|$ independent scenario-based models M_l^ω $\forall\ l \in L_2$
Sequential version:
Solve sequentially the $|\Omega|$ independent scenario-based models M_l^ω $\forall l \in L_2$.
Step 3: (Objective function value).
Compute z such that

$$z = \max(z, a_1\hat{x}_1 + b_1 y_1 + \sum_{\omega \in \Omega} w^\omega \sum_{t \in T} a_t^\omega \hat{x}_t^\omega + b_t^\omega y_t^\omega).$$

If *backward:='false'* then set *backward:='true'*, append constraint $\sum_{e \in E} x_1^e$=*num_e*-1 to model M_1^0 and go to step 1.

Computational experience and results analysis

The computational results for a set of multi-period multi-commodity networks randomly generated instances are reported as extensions of the deterministic case presented in [8] to the stochastic one. Four networks (N1,N2, N3, and N4) were generated considering 18 customers, Ω=12 scenarios and T=15 periods time horizon. Table 1 gives the models' dimensions. Its headings are: number of constraints (*m*), 0-1 variables (*n*01), continuous variables (*nc*), non-zero elements in constraint matrix (*nel*) and constraint matrix density (*den*, in %). Observe the large model dimensions of the instances considered.

Table 1: Models' dimensions

	m	*n01*	*nc*	*nel*	*den*
N1	200 578	159	91 100	783 797	0.0043
N2	369 190	309	173 735	1 516 202	0.0024
N3	706 414	609	339 005	2 975 097	0.0012
N4	823 036	618	386 663	3 360043	0.0011

For each network two instances were created by using two different sets of scenario's probabilities (namely, P1 and P2) as given in Table 2. Additionally, |L|=4, where L_1 ={1,2,3} with $t^1 = t^2 = t^3 = 1$ and L_2 ={4} where t^4=15. Levels *l*=1, 2 and 3 share the first stage but differ in the entities to each they refer to, since priorities were set among them (by order: plants, distribution centres, and disassembly centres). The macro-periods are 1, 6 and 11. Note: it is assumed that the decisions on the plants selection can only be made at period 1.

Table 2: Scenario probabilities

	Sc1	Sc2	Sc3	Sc4	Sc5	Sc6	Sc7	Sc8	Sc9	Sc10	Sc11	Sc12
P1	0.01	0.01	0.03	0.10	0.10	0.10	0.15	0.25	0.25	0.15	0.05	0.04
P2	0.01	0.01	0.01	0.02	0.05	0.10	0.05	0.15	0.10	0.18	0.12	0.20

HW/SW platform: a WS with a 2 Intel Xeon E5430 266 GHz processor (4 cores each), 24 MB of RAM, *gcc* 4.9.2 as compiler, C++ code and CPLEX v12.6 as the MIP engine.

Table 3 shows the computational comparison of CPLEX and the sequential and parallel versions of the proposed algorithm, where the latter use 8 cores. The headings are: z_{CPLEX} solution value by plain using of CPLEX; z_{alg} proposed algorithm solution value, optimality *GAP* of z_{alg} versus z_{CPLEX}, such that, $GAP\% = (z_{CPLEX} - z_{alg}) / z_{CPLEX} \cdot 100$; t_{CPLEX}, t_{seqalg}, and t_{paralg} computing times (s.) for obtaining z_{CPLEX} and z_{alg} with the sequential and parallel versions of the algorithm, respectively.

Table 3: Computational results

	z_{CPLEX}	z_{alg}	*GAP %*	t_{CPLEX}	t_{seqalg}	t_{paralg}
N1P1	25 739.9	25 739.9	0	25 653	6868	**2565**
N1P2	15 307.5	15 307.5	0	11 156	7788	**2778**
N2P1	210 684.5	210 684.5	0	**400**	2568	1085
N2P2	181 182.7	186 182.7	0	**170**	5301	3036
N3P1	-95 742.0	-95 742.0	0	768	1044	**272**
N3P2	-110 352.0	-110 352.0	0	457	1131	**342**
N4P1	221 680.7	221 680.7	0	**207**	1921	419
N4P2	151 899.0	151 899.0	0	**175**	1369	354

Observe that in all instances the algorithm produced the optimal solutions. It proved to be extremely effective in half of the instances, reducing the computing time up to one order of magnitude. On the other hand, CPLEX requires smaller computing times for those instances where the optimum is found at the very first B&B nodes, which is not very frequent in real world instances. Two major facts should be stressed. First, in all networks the parallel version of the algorithm achieved an impressive computing time reduction with respect to the sequential one, varying from 43 up to 78%. Second, the MIP problems solved at the first stage (that, by construction, is not parallelizable) are the hardest ones.

Conclusions

We have studied the performance of the sequential and parallel versions of a useful heuristic algorithm for the solution of a two-stage stochastic mixed 0-1 model of a closed loop supply chains design and operation planning problem in a dynamic setting where uncertainties appear any-where.

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