The role of teachers' knowledge and the use of different technologies in the classroom

Maria do Carmo Botelho[1] and Helena Rocha[2]

[1] Universidade NOVA de Lisboa, Faculdade de Ciências e Tecnologia, Lisboa, Portugal; mcb17696@campus.fct.unl.pt

[2] CICS NOVA, Universidade NOVA de Lisboa, Faculdade de Ciências e Tecnologia, Lisboa Portugal; hcr@fct.unl.pt

The potential of technology to enhance the learning of mathematics is widely acknowledged. However, realizing this potential requires teachers to possess the knowledge to effectively integrate it into their teaching practices. This study aims to characterize the mathematics teacher's knowledge when using different technologies in teaching and learning. It also aims to study the existence of relationships between the specific domains of the Knowledge for Teaching Mathematics with Technology – KTMT and the integration of graphing calculator and Excel. The methodology adopted is qualitative with an interpretative approach, using a case study of Mathematics teacher in the 11th grade (16-17 years old) with extensive experience in the use of technology. This study shows that different KTMT knowledge is mobilized, according to the technology chosen and the specific characteristics of each of the technologies used influenced the teacher's pedagogical choices.

Keywords: Technology integration, teacher knowledge, KTMT, Mathematics

Introduction

The use of technology in mathematics teaching has led several authors to investigate the specific knowledge needed to teach mathematics with technology (Clark-Wilson et al., 2019; Drijvers, 2019; Rocha, 2020a, 2020b), but few focus on teachers' knowledge to support students in the transition between different technologies (Bretscher, 2022). In order to create and implement learning environments, teachers must have knowledge not only of the mathematics they teach, but also of the technological tools available and their potential for exploring mathematical ideas, as well as the pedagogical aspects related to teaching and learning the subject (Kim, 2018). Regarding the knowledge required to integrate technology into professional practice, few studies address the contribution of the use of different technologies to teachers' professional development and how the different domains of their knowledge are mobilized (Rocha, 2020b). This research aims to characterize the knowledge of mathematics teachers when using different technologies in teaching and learning, and to study the existence of relationships between the specific domains of the Knowledge for Teaching Mathematics with Technology - KTMT model (Rocha, 2020b) and the integration of the chosen technology. The aim is to answer the following research questions:

(1) Which domains of KTMT are emphasized when integrating technology into professional practice?

(2) What differences are there in the knowledge mobilized by the teacher when the technology used changes?

Theoretical background

The theoretical framework of this study focuses on KTMT model (Rocha, 2022) (Figure 1), as it integrates research focused on professional knowledge and the use of technology into a single model. The KTMT considers Mathematics (MK), Teaching, and Learning, Curriculum, and Technology (TK) as the base domains of knowledge.

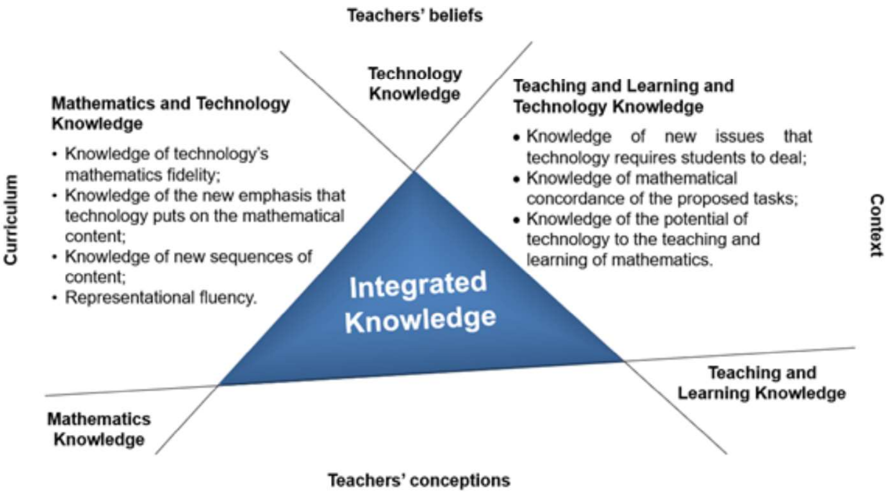

Figure 1: KTMT model (Rocha, 2022)

The mathematics base domain (MK) includes knowledge of concepts, theories and procedures of the disciplinary area. It also involves knowledge of rules of evidence and of the nature of mathematics. The technology base domain involves the capacity to operate with certain technology and essentially consists of knowing how it works and what it does. The teaching and learning base domain involves knowledge about how students think and learn, and includes an understanding of the processes usually used by students, common difficulties associated with specific content and the ability to anticipate these problems and solve them. It also includes the choices made by teachers in the different phases of teaching, involving knowledge of the sequence of activities, the different types of tasks and the way students work. Particularly valuing knowledge developed from more than one of the base domains, KTMT includes two sets of inter-domain knowledge: Mathematics and Technology Knowledge – MTK and Teaching and Learning and Technology Knowledge – TLTK. MTK focuses on how technology influences mathematics and TLTK includes how technology affects or influences teaching and learning, reinforcing or restricting certain approaches.

The strategies used by teachers to integrate different technologies into their lessons can be based on mathematical understanding or on the specific technical procedures of each technology (Botelho & Rocha, 2022). However, as highlighted by Bretscher (2022, p. 2522):

> Technology as a teaching resource for mathematics needs to be simultaneously both visible, so that it can be noticed and used in the practice of teaching school mathematics, and invisible so that attention is focused on the subject matter, teaching mathematics to pupils. The particularities of using a specific technology to teach mathematics influences the mathematics that can be taught.

Methodology

Given the aim of this study, the methodology chosen for this research is qualitative and interpretative. We used the case study of a high school mathematics teacher, whom we will refer to as Camila. She is a teacher with over thirty years' professional experience who is very familiar with using different technologies to teach mathematics, aspects that are considered particularly influential in the integration of technology (Handal et al., 2011). The data collection took place in her 11th grade class (16-17 years old), in lessons where the study of sequences using technology had been planned. As the study concerned the integration of technology by teachers into their teaching practice, and with the aim of characterizing mathematics teachers' knowledge when using different technologies, the research had a strong component of observation of mathematics teaching practice in a classroom environment. Semi-structured interviews and document analysis were also used to collect data. All the interviews and all the lessons observed were audio-recorded. Based on the tasks proposed by the teachers and in line with the nature of the study, data analysis was guided by the research questions, focusing on the use of the graphing calculator (GC) and *Excel*. For the data analysis, the moments in which different domains of KTMT were mobilized during the use of the different technologies were identified and analyzed using content analysis with special attention to the knowledge domains: TK, TLTK and MTK. The data were analyzed with the intention of identifying the utilization of knowledge associated with each of these knowledge domains. In instances where moments addressed multiple domains, thorough examination was conducted to determine which domain appeared to be predominant.

Results

In line with Bretscher (2022), Camila believes that technology must has to be a resource in her lessons, without it becoming the central aspect, taking attention away from the mathematics being taught. By integrating GCs and *Excel* into the two tasks, the teacher wants the students to become familiar with the tools provided by these two technologies. Considering that students are more familiar with the use of tables in Excel compared to GC (Ruthven & Henessy, 2002), Camila chooses to use this technology in the first task.

The task with *Excel*

At the beginning of the study of sequences, Camila tried to involve the students in an exploration of monotony and the existence of upper and lower bounds. She proposed the study of the monotony of a sequence, using the students' knowledge about the monotony of a function. Using an *Excel* spreadsheet, she proposed that the class observe the graphical and tabular representation (MTK). Having given the algebraic expression of the general term of the sequence $a_n = \frac{n+1}{3n-2}$, together with the class, the teacher built a table with the first 100 terms of a_n:

Camila: (...) what we're going to do here is visualize the behavior of a sequence by graphing, for example, the first 100 terms. (...) so here in cell two, we're going to put the values of n, because it's a natural variable, but we're not going to write them one by one, we're going to enter a formula. (...) In the next column we will insert the general expression of a_n in B3, using the Equation tool. (...)

Initially, the guidance given to the students was specific to this technology (TK), as the students had little knowledge of the tools available in *Excel.* Although all the students were using an *Excel* spreadsheet, some had to use the online version. For these students, the teacher had to give more technical instructions (TK), as some tools were not available in the online version:

Camila: For students who are using *Excel* on Google, you don't have the Equation menu, but no problem, just write the expression directly in the cell, using the brackets and the slash.

Projecting the table with only the first nine terms of a_n (Figure 2), Camila begins by asking if it is possible to conclude from the table whether the sequence is monotonic and if there is any value it is close to. One of the students replies that the sequence is decreasing, and the smallest term is 0.4. Camila then suggests showing the first 100 terms of the sequence in the table so that the students can check whether their colleague's answer was correct (MTK):

Camila: Is it possible, with this table, to say that the sequence is monotone?

Student1: Yes, it's decreasing.
Camila: The first term is 2 and we can see that the terms are decreasing. But with these 9 terms, can we say what value the terms of the sequence will approach?
Student1: Yes, the lowest value is 0.4.
Camila: Check the table with 100 terms to see if your colleague's answer is correct.
Student2: No, with n=100, the value is 0.33893.

After looking at the table with the first 100 terms of a_n the teacher provides technical guidance on the tools available in *Excel*, combining knowledge of the content of sequences (MTK) to construct the graph of a_n (Figure 3).

	A	B
1		
2	n	a_n
3	1	2,00000
4	2	0,75000
5	3	0,57143
6	4	0,50000
7	5	0,46154
8	6	0,43750
9	7	0,42105
10	8	0,40909
11	9	0,40000

Figure 3: Table with the first 9 terms of the sequence

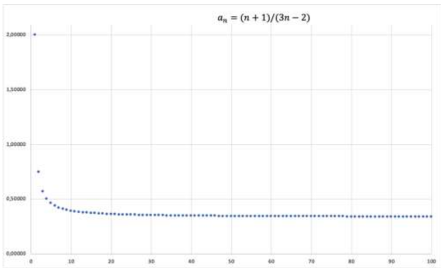

Figure 2: Graphical representation of sequence in Excel

With the graphical representation, the teacher concluded with the class, as they had already observed from the table, that the sequence was decreasing. The automatic display of the graph in a visualization window appropriate to the situation did not require Camila to make any comment on this aspect.

The teacher also wanted the students to conclude, by looking at the graph, that the sequence was bounded (MTK). She began by saying that a_n is monotonically decreasing and that its first term is two, which allowed the students to conclude that the sequence was upper bounded. However, finding a value for the lower limit by looking at the graph was more difficult, as there was no single value. The teacher therefore opted to use the table again, so that the students could see that the terms of the sequence are always positive (MTK):

Student: It [sequence] has no values greater than 2.
Camila: So, can we say that 2 is the upper bound of the sequence?
Student1: Yes, because the other values are below 2.
Camila: We've already seen that the sequence is decreasing, and the first term is 2, so it [sequence] is bounded from above. Can anyone say what a lower bound is?
Student2: No, because they're always smaller.

Camila: Let's go back to the table. We can see, for example, up to the term of order 200. What's going on? Notice that the terms of a_n are always positive, they are never less than 0. So, we can say that 0 is a lower bound of the sequence.

Camila refers to the fact that, with this technology, it is not necessary for students to adjust the viewing window to obtain a graphical representation, as the main deciding factor in choosing the technology used (TLTK):

Camila: I chose to use *Excel* to do this task, because with the calculator they had to adjust the viewing window (...). I wanted them to see the 100 terms in the table and then see the graph. With the calculator it's not so easy to see so many terms in the table, I considered that, but no, with the GC it took much more time. (...) The central aspect of my choice was the viewing window.

In this task, she did not intend to pay attention to the display window, since with *Excel* the window is automatically adjusted. The knowledge of the impact of the viewing window on the aspect of the graph is a decisive factor when choosing the technology (MTK):

Camila: I chose to use *Excel* because I want them to focus on studying monotony [of sequence] through the table and graph. In this first task, I don't want them to have to change the window, because I know it's still not easy for them to understand how and when to change the window. I want them to be able to see the terms of the sequence straight away without having to change the window.

The task with graphing calculator

In the second task, the teacher wanted the students to study monotony and check whether it was bounded, using GC. This was the first time *Recursion* tool had been used in class, Camila gives specific technical guidelines and exemplifies them so that the students can repeat the procedures in their GCs (TK). In the class, only one student did not have a Casio *fx-CG* and so the guidance given was only for this technology. Given the sequence $b_n = \left(\frac{2}{3}\right)^n$, the teacher explains how to insert its expression (Figure 4) into the GC (TK), starting by defining the type of expression for the general term (Figure 5) (MTK). Next, by presenting the table with the first four terms of b_n (Figure 6), she asks the class about the monotony of sequence and whether sequence is bounded:

Camila: Looking at these four terms in the table, can we conclude what the monotony of the sequence is?
Student: It's monotonically decreasing.
Camila: Is it [sequence] bounded?
Student: 0.6666 is the biggest but we can't tell if it will stop at 0.1975.
Camila: So, let's look at the graph.

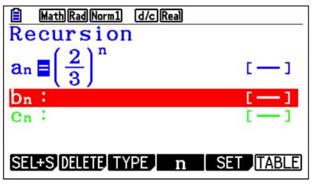

Figure 4: General term of b_n

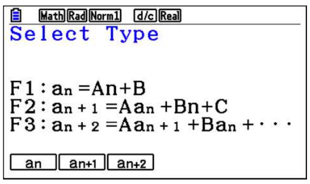

Figure 5: General Term Type Selection Menu

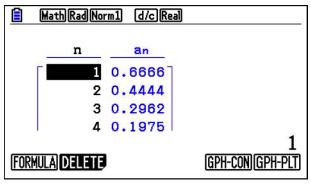

Figure 6: Table with the first four terms of the b_n

With the sequence graph displayed in a standard viewing window (Figure 7), Camila gives guidance on how to change it, discussing the appropriate parameter values with the class (MTK). By changing the window, the students are able to see the graphical representation of the five terms of b_n (Figure 8).

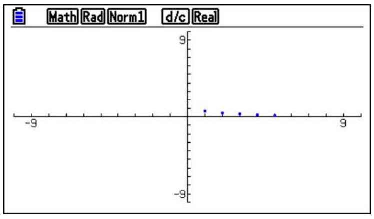

Figure 7: Graph obtained with a standard viewing window

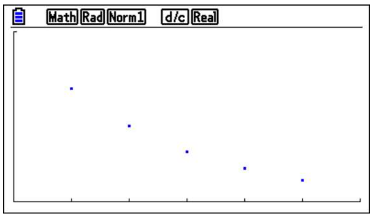

Figure 8: Graph obtained with the appropriate viewing window

The need to change and adjust the display window did not arise when studying sequence a_n in *Excel*, since the graph in this technology is automatically adjusted to the data entered in the table. Recognizing that this is a difficult process when using GC, Camila begins by presenting the graph in a standard window, with the intention of creating in the students the need to adjust it, so that they can find a more suitable viewing window:

Camila: My idea was for them to first see the graph [of b_n] in the standard window and from there, make them realize that the window had to be adjusted. After seeing the first graph, it was easy for them to find the right values for the viewing window (...).

When asked by the students about the existence of a lower bound, Camila explains that since the terms of the sequence are powers of a positive base and a natural exponent, the terms are all positive. She then leads the class to conclude that the sequence is also bounded from below, and that 0 is a possible lower bound, but not the only one (MK).

Discussion and conclusion

The KTMT model seeks to combine the domains of knowledge considered relevant in the research with the specificities of technology. It highlights the importance of the influence of technology on the different domains of knowledge, emphasizing the need to develop MTK and TLTK to effectively integrate technology into professional practice. It is therefore considered that these inter-domain knowledge is essential as a basis for analyzing teacher knowledge (Rocha, 2022). Although the pedagogical options that technology allows are recognized (Clark-Wilson et al., 2020; Bretscher, 2022; Rocha, 2020a, 2020b), the specific characteristics of each of the technologies used influenced the teacher's pedagogical choices when solving the tasks (Ruthven, 2009). The knowledge of the students' difficulties in finding a suitable viewing window (TLTK) was decisive in the choice of technology that the teacher used in the first task. The teacher decided to use technology where less technical knowledge had to be mobilized, and where there was no need to adjust the display window. The teacher's knowledge of the impact of the visualization window on the appearance of the graph (MTK) leads her to consider that the tasks have different degrees of difficulty, depending on the technology chosen. Teacher’s guidance differed for the two technologies. With GC, her guidance was necessary for the articulation between the different menus and the adjustment of the viewing window to obtain the graph. By explaining how the different menus work, the teacher emphasizes more mathematical aspects. It was also found that the integration of these technologies had an impact on the way the mathematical content was taught (Bretscher, 2022). The use of the two technologies promoted the mobilization of different domains of knowledge, as highlighted by Rocha (2020a, 2020b). It was observed that the teacher mobilized MTK, TLTK, TK and MK and, in some situations, these domains were mobilized in an integrated way. However, there are no differences in the way they are mobilized according to the technology used. In both tasks, the integration of technology into her professional practice reveals the existence of an interconnection between MK, TK and TLTK, as considered in KTMT. By adapting her strategies to different technologies (*Excel* and GC), Camila demonstrates that she is aware of the impact of technology on mathematics teaching. In this sense, she gives specific guidance and adjusts approaches to the tools provided by each of the technologies. It is this knowledge of the most appropriate strategies (TK) for each situation that influences the guidance given by the teacher (TLTK) (Rocha, 2020a). The technical guidance given by the teacher on using *Excel*, reflects an understanding of the impact of this technology on teaching and learning, mobilizing TLTK for this purpose. The need to change the viewing window in the GC and the subsequent explanation of this change also involved the mobilization of TLTK. The teacher guides the students in handling the technology, not only to improve their visual understanding of the graphs, but also to reconcile the mathematical knowledge that they have learned. Camila also reveals an understanding of the impact that the visualization window has on the appearance of the graph, which allows her to analyze the degree of difficulty associated with each of the tasks (MTK) (Rocha, 2020a). In both technologies, the teacher helps the students to understand mathematics by articulating the tabular representation with the graphical representation. However, there is a greater mobilization of MTK with the GC. In this technology, the exploration of a graph and the need to adjust the viewing window provides a more exploratory approach to the study of the sequence monotony. It should also be noted that the combination of algebraic, graphical and tabular representation on the GC allowed for a more dynamic strategy. In summary, the pedagogical approaches adopted by Camila reflect an advanced level of development of her KTMT. The integration of different technologies has allowed her to mobilize different domains of knowledge. This integration demonstrates the teacher's deep inter-domain knowledge (MTK and TLTK), in line with the principles of KTMT. We recognize that this study is limited by its analysis of only two technologies and does not cover the diversity that exists. Future studies should be designed to observe the integration of a greater number of technologies with in-service and pre-service teachers.

Acknowledgment: This work is supported by national funds through the FCT – Foundation for Science and Technology, I.P., which granted a PhD scholarship for the first author (2021.04983. BD) and funded the project TecTeachers (https://doi.org/10.54499/2022.03892.PTDC).

References

Botelho, M. C., & Rocha, H. (2022). O conhecimento profissional do professor de matemática na integração de diferentes tecnologias. In A. Silvestre, C. Torres, H. Pinto, J. Cabral & M. Rodrigues (Eds.), *Proceedings of the XXXII* Seminário de Investigação em Educação Matemática (pp. 158-161). Associação de Professores de Matemática.

Bretscher, N. (2022). Teachers' knowledge for supporting transitions between dynamic digital technologies and static environments. In J. Hodgen, E. Geraniou, G. Bolondi, & F. Ferretti (Eds.), *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education (CERME12)* (pp. 2516-2523). Free University of Bozen-Bolzano and ERME. https://hal.science/hal- 0374748

Clark-Wilson, A., Robutti, O., & Thomas, M. (2020). Teaching with digital technology. *ZDM, 52*(7), 1223-1242. https://doi.org/10.1007/s11858-020-01196-0

Drijvers, P. (2019). Head in the clouds, feet on the ground – A realistic view on using digital tools in mathematics education. In A. Büchter, M. Glade, R. Herold-Blasius, M. Klinger, F. Schacht, & P. Scherer (Eds.), *Vielfältige Zugänge zum Mathematikunterricht* (pp. 163–176). Springer. https://doi.org/10.1007/978-3-658-24292-3_12

Handal, B., Cavanagh, M., Wood, L., & Petocz, P. (2011). Factors leading to the adoption of a learning technology: The case of graphics calculators. *Australasian Journal of Educational Technology*, *27*(2), 343-360. https://doi.org/10.14742/ajet.974

Kim, S. (2018). Technological, pedagogical, and content knowledge (TPACK) and beliefs of preservice secondary mathematics teachers: Examining the relationships. *Eurasia Journal of Mathematics, Science and Technology Education, 14*(10), 1-24. https://doi.org/10.29333/ejmste/93179

Rocha, H. (2020a). Graphical representation of functions using technology: a window to teacher knowledge. *Teaching Mathematics and its Applications*, *39*(2), 105-126. https://doi.org/10.1093/teamat/hrz011

Rocha, H. (2020b). Using tasks to develop pre-service teachers' knowledge for teaching mathematics with digital technology. *ZDM, 52*(7), 1381-1396. https://doi.org/10.1007/s11858-020-01195-1

Rocha, H. (2022). Pre-service teachers' knowledge and the use of different technologies to teach Mathematics. In A. Mesquita, A. Abreu, & J. V. Carvalho (Eds.), Perspectives and Trends in Education and Technology (pp. 505-515). Springer. https://doi.org/10.1007/978-981-16-5063-5_42

Ruthven, K., & Hennessy, S. (2002). A practitioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics, 49* (1), 47-88.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Education and Didactique, 3*(1), 131-159. https://doi.org/10.4000/educationdidactique.434