Teachers as task designers in the digital age: teaching using technology

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The aim of the paper is to present and analyse the case of one teacher attempting to introduce his students to fractals using digital technology. His task design process has been made explicit through the writing of a storyboard. It has been analysed in order to focus on the stages of the process, identifying prominent elements in it by using the knowledge quartet framework. Results can be useful to inform teacher educators about his needs with respect to the development of his ability in task design. The importance of this aspect, particularly worth of note in the digital age in which teachers have many opportunities to access teaching resources online, has been amplified by the constraints to which educational systems have been subjected during the Covid-19 pandemic emergency.

Keywords: Task design, digital technology, teacher knowledge, teacher professional development.

INTRODUCTION

Digital technology is usually pointed as having the potential to promote the students' development of mathematical thinking. However, according to Thomas and Lin (2013), this development is more a consequence of the tasks proposed to the students and of the way these tasks explore the potentialities of the technology, than of the technology alone. Nevertheless, designing tasks that enhance the potential of technology is a complex and difficult achievement (Joubert, 2017) and even a major pedagogical activity (Leung, 2017). There is the need to design tasks promoting mathematical learning and understanding, and which take advantage of technology. This can challenge the curriculum and the teaching trajectories, changing the more traditional approaches. Often, mathematical tasks aim at achieving results or answers, emphasizing procedural skills instead of promoting conceptual understanding and the development of problem solving competencies. The use of technology changes the way of having access to results and facilitates a focus on conceptual understanding (Rocha, 2020). However, this might result in a need to new prerequisites for designing tasks (Olsson, 2019).

This paper presents and analyses a case of one teacher attempting to integrate digital technology in his teaching. We intend to reflect on how he was engaged in the development of digital technology rich learning trajectory concerning fractals. Fractals has been chosen for their potential to address several mathematical contents, and also because it is possible to find on-line many different kinds of resources concerning them. We address the research question: what are prominent elements characterising stages of the teacher’s task design process that can be identified? Although the paper concerns a single case study and a specific mathematical topic, the findings can be useful to inform teacher educators about teachers' needs related to the development of their task design ability. The importance of this aspect, particularly in the digital age in which teachers have many opportunities to access teaching resources online, has been amplified by the constraints to which educational systems have been subjected during the Covid-19 pandemic emergency.

THEORETICAL FRAMEWORK

Teachers always design tasks in order to promote their students learning. According to Leung (2017, p. 4), "mathematics task design can be thought of as designing activities situated in pedagogical environments that provide boundaries within which students engage in doing mathematics leading to the construction of mathematical knowledge". The decisions teachers take during this process of creation are guided by the learning goals they define for their students (e.g., planning an exploration activity or a moment to practice). This process becomes more complex when digital technologies are part of it and, in these circumstances, for students to take advantage of all the potential provided by technology, the tasks should require them to explore, reconstruct and explain mathematical concepts and relations (e.g., Olsson, 2019).

When referring to different types of task sequences, the global idea given by Watson et al. (2013) is that the initial tasks of a sequence somehow offer a basis for the development of mathematics knowledge needed to address the later tasks. However, most of the research focuses on isolated tasks, trying to characterize them. For example, Burkhardt and Swan (2013) give attention to the difficulty of the task, analysing the task according to different factors: complexity; unfamiliarity; mathematical procedural demand; student autonomy and level or kind of guidance. Other authors, e.g. Ponte (2005), classify tasks according to their level of difficulty but also according to their level of structure (from closed to open-ended tasks).

Also, when considering the use of digital technologies in a task, there are different roles they can assume (Rocha, 2020). Actually, technology can be integrated into a task as a way of doing part of the mathematics, as a way of allowing for exploration of a situation and the development of conjectures, or in several other roles. Laborde (2001) classifies the tasks according to the role assumed on it by technology, but in a different way: tasks that are facilitated by technology but not modified by it; tasks where technology facilitates exploration and analysis; tasks that can be done with paper and pencil, but where technology allows new approaches; tasks that cannot be accomplished without technology. In this case the focus is not so much on what technology does, but more on how the use of technology impacts the task.

According to Leung (2017), the teacher knowledge is central in the options assumed by the teacher in the process of task design. The author assumes this knowledge as a complex construct resulting from the interactions among different knowledge domains. And from these interactions Rocha (2013) emphasizes the impact of technology on the pedagogical options of the teacher and also on the mathematical content addressed.

Rowland et al. (2005) also value the teacher knowledge and highlight four dimensions of what they call quartet knowledge: foundation, transformation, connection, and contingency. While the last dimension can be described as the ability to "think on one's feet" during the contingent classroom events, the first three dimensions have been further characterized by Tanışlı et al. (2019, p. 136) in the following way: "foundation refers to a repertoire of the teacher's academic knowledge for teaching and learning mathematics including his/her beliefs regarding why mathematics is important and why it should be taught"; "transformation refers to the transformation of theoretical knowledge into practice by designing and planning pedagogical tasks in terms of choosing appropriate examples and activities for the construction of mathematical meanings"; "connection refers to the coherence of designed parts of a lesson or series of lessons through deliberately chosen activities and domain specific tasks. Such pedagogical task sequences enable students to make a connection between different concepts as well as to interplay between different representations". These dimensions give to the conceptualization a close connection to the teachers' practice, particularly suitable for this study.

METHODOLOGY AND CONTEXT

As we were interested in identifying prominent elements in the stages of the teacher task design process, we used storyboards as research tools. The storyboard we analysed in this paper was written by an Italian mathematics teacher, with a Master degree in Mathematics, working at the high-school level for 7 years. Although he was not enrolled at that moment in any research or training programme, in order to become a teacher, he was involved in a two-year teacher education program ending with an examination. Thanks to this program he acquired basic notions of mathematics education and of the use of technology in the teaching practices. He designed a teaching sequence of tasks which involves digital technology and concerns fractals, attempting to exploit the opportunity to approach them in different ways and at different levels. His task design process was made explicit through the writing of a storyboard. Excerpts of the storyboard are presented and analysed using the dimensions of the quartet knowledge framework. Although the framework was thought mainly to focus on the analysis of mathematics teaching, we believe it could be useful to develop some understanding also about the way teachers are engaged in their task design process in a context of technology integration.

RESULTS

In the storyboard analysed in this paper, the sequence of tasks was conceived to be hypothetically developed in four hours in the laboratory, so that pairs of students could share a computer. Herein, we focus on the teacher's task design process starting with the presentation of the teacher's plans related to his hypothetical task sequence.

According to what the teacher wrote in the storyboard, he starts his teaching sequence by posing a problem. The chosen problem seems not to be immediately connected with fractals, but it is pivotal in the development of the sequence. Indeed, his aim is to let emerge from the discussion that the curve "broken infinite times" presents a difference with that "broken 1000 or 100000 times". At this point he plans to show a short part of a video, explaining the property of the Roman cabbage of "remaining equal to itself" on any scale, and hence to give a definition of fractals using this property, so to introduce the concept of self-similarity.

In the next phase, he plans to introduce the students to the many examples in which nature uses fractal structures. In a brainstorming session he intends to invite students to propose possible explanations as to why nature often uses these structures. The final answer is entrusted again to a short part of a video that shows the increase of the surface in a limited volume. This makes students reflect on the "relationship between dimensions (surface – volume)". Then, he plans to show fractal constructions that can be obtained by recurrence through geometric transformations, such as the Koch curve and the Sierpinski triangle. In doing these constructions he is particularly interested in highlighting the role of the affine transformations and determining their equations:

> It will be shown how to use the self-similarity of fractals to determine the minimum number of portions "equal to themselves" that allows us to obtain the whole figure and how to apply the transformations to these parts. This will be done for the Koch curve and the Sierpinski triangle and, after having collectively identified the parts and transformations, students will be invited to write their equations.

Successively, he plans to show some tutorials –founded online– presenting the creation of ad hoc tools to reproduce the minimal part of the similarity –in order to directly involve the students, in pairs or in small groups, in the construction of fractals using GeoGebra. He intends to underline how the identification of the transformations is the basis on which the ad hoc tools are created.

> The possibility that GeoGebra gives to zoom in on portions of areas will allow students to better understand how fractals are related to the concept of infinity.

Finally, his idea is to come back to the starting problem to explain how fractals can be useful to tackle these kinds of issues.

In what follows, we show how the foundation, transformation and connection dimensions of the quartet knowledge framework can be identified in the teacher's storyboard on the task design process. The last dimension, contingency is exclusively related to the implementation in the classroom and would not be addressed here.

Foundation

The teacher identifies mathematical content that can be addressed using fractals. This can be seen as characterizing his knowledge with respect to the foundation dimension. For example, he writes:

> Fractals involve different mathematical concepts (and not only): geometry (in a broad sense), proportions, geometric transformations, concept of dimension, arithmetic, trigonometry, successions, functions, limits and convergences, set theory, logic, ...

> The intention in this case is not to be limited to purely exposition: the presentation of these mathematical objects with the playful, artistic and anecdotal aspects, in my view, could be combined with the use of fractals as application and study of geometric transformations in the Cartesian plane. On this aspect, therefore, my teaching intervention intends to be more mathematically punctual and "operational". Moreover, from a more cultural point of view, I would like to take this opportunity for a (possibly different) reflection on the concept of infinity and its implications with mathematics.

He also underlines that the Internet offers a surprisingly large variety of software to create fractals, but he considers the idea of generating fractals to come out of the purpose of a limited educational intervention aimed at a mathematical learning goal. However, in some software he sees aspects that can be useful from the didactical point of view. For example, he found a software that has a rich gallery of interactive fractals: "you can zoom in (in fact the program recompiles always guaranteeing excellent definition and richness of details), select portions of the image, rotate them, vary the colours of the convergence sets". He recognises in this interactive functionality a didactical potentiality.

The foundation dimension of the teacher's knowledge can also be seen in the way he looks for online resources and in the comments he writes concerning the choice of the resources with respect to his beliefs regarding mathematics teaching. Two examples are given below:

> - This is one of the first websites I found (of some interest). The most interesting part is a section in which it proposes programs written in BASIC that should build fractals showing their iterations. This thing is of potential interest, but in addition to the fact that I already had doubts about the usefulness of running a code that for students means nothing (not knowing the language), honestly by doing the tests I was not even able to run them.
>
> - This website is absolutely pertinent to the aims of the educational intervention that I have in mind, given that it emphasizes exactly the related transformations. The website is divided into three parts: related transformations in the plan (this part should be a prerequisite for the students, but it is convenient that it is also present on the website); fractal geometry (it is the main and most interesting part for my purposes); insights (especially centred on the relationship between fractals - golden section - spirals).

Transformation

When the teacher describes the way he analysed the resources he found online, he makes explicit some assumptions which reveal his way of perceiving the knowledge in action. As the quote below shows, his analysis of the resources reveals not only his pedagogical point of view but also the mathematical one. Concerning some of the resources he took into consideration he makes the following kind of comments:

> - This video presents fractals giving excellent ideas and using a captivating video editing.
>
> - The intent of the video seems to me to bring out the mathematics that "underlies" the fractals in a way that can be used by non-professionals, but the speed of the exposure

makes, in my opinion, the whole video inadequate for students who, in addition to being captivated, should also learn something.

- Some issues that can be tackled by high school students are completely left out or excessively trivialised by this video.

- The attempt to keep content within a few minutes is appreciable but, dealing with so many aspects in such a short time, it does not make this video suitable even for a simple "first presentation" of the topic.

This foundation knowledge led him to do some transformation and he decided to choose small parts of videos to address specific aspects that can benefit from the visualization allowed by the video format.

He also declared the willingness to let students experiment by themselves with the construction of fractals. Taking into account that his aim was mainly to show the relationship between the related transformations and the generation of fractals, he thinks about using GeoGebra and behaving "by hand", so that students can "touch" the transformations involved and the necessary iterations.

Connection

The way the teacher structures the teaching sequence reflects choices based not only on the knowledge of structural connections within mathematics itself, but also on his pedagogical content knowledge. For instance, his foundation knowledge makes him value an approach based on problem solving as a way to involve students, and his connection knowledge makes him decide to start by what he called a "stimulus" problem. The particularity of the problem he chooses was that it could be faced by the students considering two approaches which come out as two different and conflicting answers. This was, in his hypothetical teaching sequence, the occasion to promote a brainstorming that could bring students to the need of discovering fractals. He values the relevance for the students of what is addressed (foundation knowledge), as so, he tries to lead students to recognize fractals around them. In this sense he moves from the initial (mathematical) approach towards some issues that allow students to connect fractals around them again with mathematics point of view.

The role of technology in the teacher task design

A focus on the role of technology in the task design process of the teacher can be added to the analysis in terms of the quartet knowledge framework. This would add some understanding to the case of a task design process involving technology. It is possible to identify some stages on the teacher's task design that are directly related to technology and grounded in the teacher's foundation knowledge: looking for digital materials concerning fractals; choosing some of the most "interesting"; and reflecting on the potentialities of them. Concerning transformation knowledge, some other stages can be identified: focusing on a mathematical content that can be mediated by the use of them; and recognising aspects of the content that can or cannot be taken into consideration when using them. Finally, the building of a sequence of tasks using the chosen digital material, can be seen as the final stage grounded in the teacher's connection knowledge.

CONCLUSION

Nowadays, there are many resources available online. This circumstance increases the relevance of the teachers' knowledge to allow them to be able to explore and use resources to promote learning. With the aim to identify prominent elements characterising the stages in the task design process, our conclusions, based on the quartet knowledge framework, point to a significant impact of the teachers' foundation knowledge. Although we have only analysed the task design of one teacher, as far the presented case study is concerned, it is this knowledge that guides the initial options of the teacher, defining what he considers relevant to use and what he does not. This is the starting point for modifications to the resources, guided by the teacher's transformation knowledge. And this is the source for the alignment of the sequence of tasks, where the connection knowledge defines the learning trajectory proposed to the students. The prominent elements characterising the stages related with the use of technology, as they emerged through the analysis, can be grounded in all the three dimensions. However, our findings highlight that foundation knowledge has a strong influence over the initial choice of digital technologies and over the reflection about their potentialities. This could be useful to inform teacher educators about teachers' needs with respect to the development of their ability in task design. In particular, in order to promote a rich selection of digital technologies, it seems important, within the teachers training programs (initial and continuous), to give attention to the development of foundation knowledge. In this way it will be possible to develop a deep integration of digital technologies on teachers' practice. Nevertheless, this requires attention also to the transformation and connection knowledge of the teachers, in order to transform technology into an irreplaceable part of the tasks (according to the view presented by Laborde, 2001), where tasks are changed by the technology and cannot be implemented without it.

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REFERENCES

Burkhardt, H. & Swan, M. (2013). Task design for systemic improvement: principles and frameworks. In C. Margolinas (Ed.), *Task design in Mathematics Education – Proceedings of ICMI Study 22* (pp. 431-440). ICMI.

Joubert, M. (2017). Revisiting theory for the design of tasks: Special considerations for digital environments. In A. Leung & A. Baccaglini-Frank (Eds.), *Digital* *technologies in designing mathematics education tasks - Potential and pitfalls* (pp. 17-40). Springer.

Laborde, C. (2001). Integration of technology in design of geometry tasks with Cabri-geometry. *International Journal of Computers for Mathematical Learning*, 6, 283-317.

Leung, A. (2017). Exploring techno-pedagogic task design in the mathematics classroom. In A. Leung & A. Baccaglini-Frank (Eds.), *Digital technologies in designing mathematics education tasks - Potential and pitfalls* (pp. 3-16). Springer.

Olsson, J. (2019). Relations between task design and students' utilization of GeoGebra. *Digital Experiences in Mathematics Education*, 5, 223-251.

Ponte, J. (2005). Gestão curricular em Matemática. In GTI (Eds.), *O professor e o desenvolvimento curricular* (pp. 11-34). APM.

Rocha, H. (2013). Knowledge for Teaching Mathematics with Technology – a new framework of teacher knowledge. In A. Lindmeier & A. Heinze (Eds.), *Proceedings of the 37th PME* (vol. 4, pp. 105–112). PME.

Rocha, H. (2020). Graphical representation of functions using technology: a window to teacher knowledge. *Teaching Mathematics and its Applications*, 39(2), 105-126.

Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: the knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education*, 8, 255-281.

Tanışlı, D., Köse, N., & Turgut, M. (2019). Professional development through a webbased portal: The progress of mathematics teachers teaching algebra based on hypothetical learning trajectories. In U. T. Jankvist, M. van den Heuvel-Panhuizen, & M. Veldhuis (Eds.), *Proceedings of CERME 11* (pp. 2741-2748). Freudenthal Group & Freudenthal Institute, Utrecht University and ERME.

Thomas, M. & Lin, C. (2013). Designing tasks for use with digital technology. In C. Margolinas (Ed.), Task design in Mathematics Education – *Proceedings of ICMI Study 22* (pp. 109-118). ICMI.

Watson, A., et al. (2013). Introduction. In C. Margolinas (Ed.), Task design in Mathematics Education – *Proceedings of ICMI Study 22* (pp. 7-14). ICMI.