ESTIMATION OF THE TENSILE FORCE IN THE STAY-CABLES OF SALGUEIRO MAIA BRIDGE USING AMBIENT VIBRATION TESTS

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Abstract

Nowadays, the estimation of tensile forces in stay-cables or external tendons of Civil Engineering structures is required during periodic inspections. Among the available procedures, the ambient vibration testing (AVT) is recognized, in the scientific community, as an expedited technique. However, its accuracy is closely related to precise estimation of three key parameters that characterize the dynamic response of the cable, namely its mass, free length and bending stiffness. While the cable mass is, usually, readily available, a certain uncertainty is associated to the cable free length and bending stiffness, hindering the accuracy of the assessed cable forces. This paper presents a practically applicable optimization procedure that allows, for a given free length, to simultaneously estimate both the installed tensile force and the cable bending stiffness. The application of this methodology is shown on the stay-cables of Salgueiro Maia Bridge, crossing the Tejo River in Santarém, Portugal.

INTRODUCTION

In the last decades, the estimation of the tensile forces installed in stay-cables or exterior tendons of Civil Engineering structures through AVT has been widely studied. The growing interest of engineering practitioners in this technique is explained by the satisfactory results that can be obtained using this expedite procedure. This procedure can be useful in both construction and service situations, in structures without pre-installed instrumentation for the real-time monitoring of the installed forces in cables [1, 2, 3].

However, vibration measurements conducted and reported in the literature have shown that, sometimes, the achieved accuracy was not acceptable. To produce accurate estimates, for the installed tensile forces, this methodology requires that certain characteristic cable parameters (cable mass, free length and bending stiffness) are defined with the lowest degree of uncertainty possible. While the cable mass is usually available, the definition of the cable free length and bending stiffness is often a challenging problem. In an attempt to reduce the error associated to the above mentioned uncertainties, the present paper presents a possible definition for a multi-variable optimization problem, used to simultaneously estimate the combination of cable tensile force, free length and bending stiffness that best match the cable measured modal properties. The application of this methodology is exemplified on eighteen stay-cables of Salgueiro Maia Bridge, shown in Figure 1, crossing the Tejo River in Santarém, Portugal.

Figure 1 - Salgueiro Maia Bridge over the Tejo River

THEORETICAL FORMULATION

The estimation of the installed tensile force in stay-cables and exterior tendons through AVT is usually based on the vibrating string theory [4]. This allows to relate the natural frequencies of the vibration modes *i* of a string with its tensile force *N*.

$$\frac{\quad}{\quad}\sqrt{\frac{\quad}{\quad}} \qquad (1)$$

Equation (1), which describes this theory, is quite simple and its deduction is easy to understand. However, it implies simplifications that may not be admissible when applied to the case of Civil Engineering structures: although it considers the cable free vibration length *L* and its distributed mass per unit length *m*, it disregards the cable bending stiffness and its curvature associated to self-weight.

It is already known [5, 6] that the vibrating string theory is no longer valid when the cables are too short. For these cases, the influence of the bending stiffness (*EI*) is no longer negligible and the behaviour of the cables is therefore close to that of a beam clamped at both ends. The natural frequencies of the clamped beam were deduced by Morse and Ingard [4] and include the effects of *EI*. Subsequently, Mehrabi and Tabatabai [7] developed a formulation that considers not only the effects of bending stiffness but also the effects of the curvature of the cables due to self-weight.

In this former formulation (which is the one adopted in the present study), the bending stiffness, which mainly affects the upper order vibration modes, is introduced through the dimensionless bending stiffness ζ given by

(2)

To consider the effects of the typical geometric nonlinearity of cable problems, the Young modulus of elasticity *E* must be corrected to the valued given by Ernest [8],

(3)

where *E* and *γ* are the Young modulus of elasticity and the specific weight of the material, respectively, the horizontal projection of the free length of the cable and *σ* its installed stress.

In what respects the effects of the cable sag, they essentially affect the first mode of vibration and are taken into account through the Irvine parameter [9], defined as

(4)

where,

(5) and (6)

The corrective formulation derived by Mehrabi and Tabatabai [7] is then written as a function of and , and is given by

(7)

where,

(8) , (9) and (10)

UNCERTAINITIES ASSOCIATED TO FREE LENGTH AND BENDING STIFFNESS

When equation (7) is to be applied in practice, the definition of the free length *L* and the bending stiffness *EI* is challenging and usually associated to a certain degree of uncertainty.

The cable free length of vibration, *L*, is defined as the distance between the two modal nodes close to the cable ends. In the case of stay-cable, the location of these nodes is between the strands deviator (often inside protective jackets) and the anchor plate where the strands are fixed. The associated uncertainty can reach significant values (±3% in the present case study) and hinder the accuracy of the estimated tensile force. A possible solution to minimize the error in the definition of the free length is to perform a modal identification of higher order modes and to measure the length between the nodes of higher order modes close to the anchor [1]. However, this is a burdensome procedure and is seldom used in practice, as it requires measurements on several points along the cable. It was also not considered in the present study.

In what respects the definition of the bending stiffness, *EI*, it is necessary to take into account that the relative slip between the strands of the cable affects the cable moment of inertia, *I*. Therefore, the value of *I* will vary between (no interaction between strands) e (monolithic cross section), defined by equations (11) and (12), respectively [10],

(11) (12)

where *n* is the number of strands, , and are the moment of inertia, the cross-section and the relative position of an individual strand of the cable. One can see that the range of possible values for *I* is considerable and therefore, estimating a reasonable value is not trivial.

OPTIMIZATION PROBLEM

In an attempt to reduce the error associated to the above uncertainties, and taking advantage of the technological evolution that currently allows the identification of a high number of natural frequencies from inexpensive AVT, a multi-variable optimization problem was defined with the objective of adjusting the theoretical natural frequencies given by equation (7) to the natural frequencies identified experimentally, by simultaneous variation of *N*, *L* and *EI*.

To better understand the way each of the optimization variables affects the natural frequencies of the cables, a sensitivity study was performed for a characteristic stay-cable of the Salgueiro Maia Bridge. Figures 2 to 4 illustrate the sensitivity of the first 15 scaled natural frequencies to cable axial force, free length and moment of inertia.

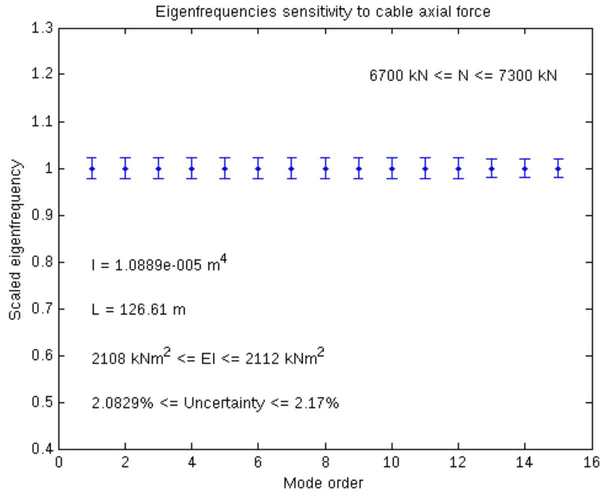

Figure 2 - Frequencies sensitivity to cable axial force

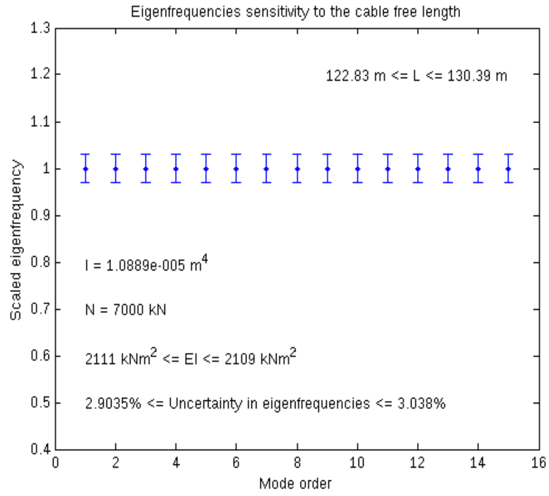

Figure 3 - Frequencies sensitivity to the cable free length

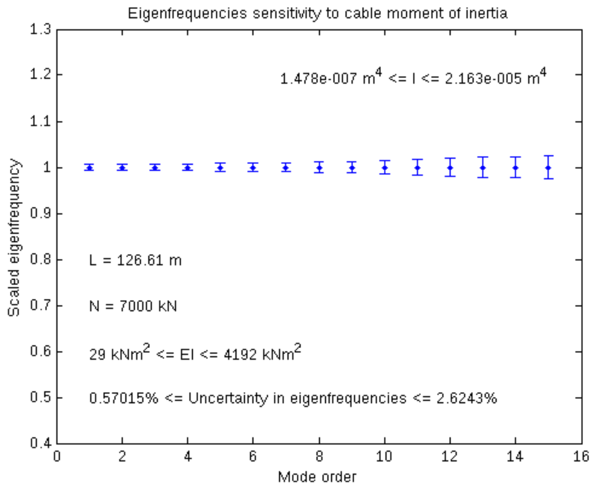

Figure 4 - Frequencies sensitivity to cable moment of inertia

Analysing the results presented in Figures 2 to 4 one can readily see that, while both the axial force *N* and the free length *L* affect evenly all the natural frequencies of the cable, the cable moment of inertia *I* mainly affects the higher modes. It can also be observed that, for similar relative variations of *N* and *L*, the free vibration length has a greater effect on the frequency value (therefore, its value is important to be known as accurately as possible). Moreover, one can see that uncertainty caused by *I* is more than 4 times bigger in the 15th frequency than the one corresponding to the 1st frequency. Therefore, it is expected the need of a large number of experimental natural frequencies for accurate estimates of the cable bending stiffness.

Based on the results of the sensitivity tests, two objective functions were defined using the experimental values of the natural frequencies of the cable, , and the theoretical counterparts defined by equation (7),

(13) (14)

Equation (13), identified as unscaled objective function, is more sensitive to higher frequencies and therefore, when used in the optimization problem will produce a solution more sensitive to the bending stiffness of the cable. Alternatively, equation (14), identified as scaled objective function is evenly sensitive to all frequencies and is expected to produce a more balanced result. Using these objective functions, the optimization problem is therefore expressed as:

(15)

The result of the optimization problem (15) is the set of and which minimize the objective function (13) or (14), for a close match between the experimental and theoretical values of the natural frequencies of the cable.

CASE STUDY: SALGUEIRO MAIA BRIDGE

The Salgueiro Maia Bridge, located in the city of Santarém, Portugal, is a motorway cable-stayed bridge with a semi-fan configuration. Inaugurated in 2000, it has a total length of 570 m, of which 486 m are suspended by 72 stay-cables, divided between two masts. The stay-cables have total lengths ranging from 31 to 131 m and are made up of 55, 61 or 73 self-protected strands, wrapped in High Density Polyethylene jackets. The present study is focused on the 18 stay-cables closest to the Santarém bank, identified as T01t to T18t, with the first being the shortest stay-cable and the last the longest.

During the experimental campaign, the vibrations of the 18 stay-cables were recorded for 10 minutes with an acquisition rate of 100 Hz, using a PCB accelerometer (model 393B04) connected to a SCXI-1530 acquisition board on a NXI-6221 National Instruments platform (see Figures 5 and 6).


Figure 5 – Positioning of the acquisition system

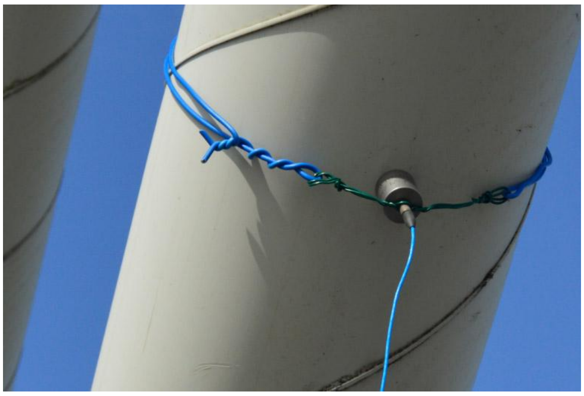
Figure 6 – Fastening of the accelerometer

In what follows, the estimation of the tensile force using the above methodology is illustrated for the T18t stay-cable. First, the first natural frequencies are estimated by performing a FFT on the signal recorded during the AVT. For this cable, the first 15 natural frequencies were readily identified, as shown in Figure 7.

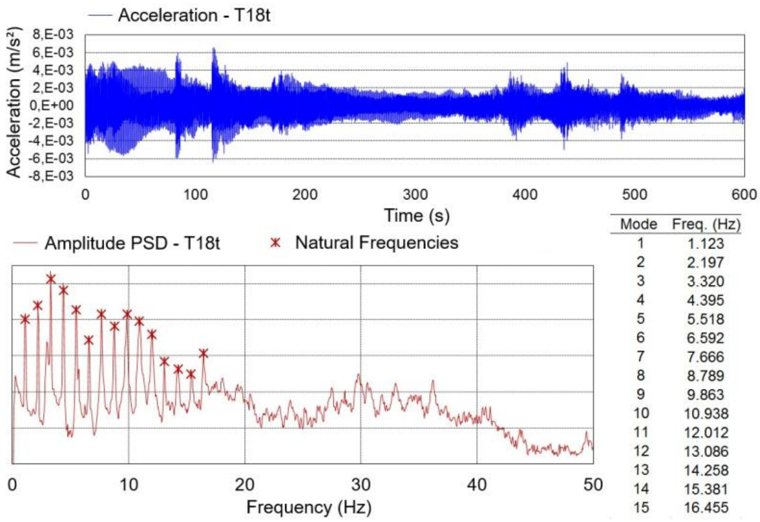

Figure 7 – Stay-cable T18t: recorded signal and modal identification

The next step is to define uncertainty intervals for *L* and *I*. The maximum allowed value for the cable free length of vibration, , defined as the distance between the anchor plates, is identified according to the design drawings of the bridge as 130.39 m. The minimum allowed value, , defined as the distance between the strands guide deviators, is also identified as 122.83 m. Therefore, the free length of vibration for this cable, *L*, can be estimated to 126.61 m with an uncertainty of ±3%. The admissible range for the cross-section moment of inertia is computed considering the 73 strands of the cable, their configuration (given in the design drawings) and equations (11) and (12), to yield and as m⁴ and m⁴, respectively.

To better understand the relation between *N*, *L* and *I* in the optimization problem, the objective functions defined by equations (13) and (14) are plotted for a constant *I* in Figures 8 and 9, and for constant *L* in Figures 10 and 11.

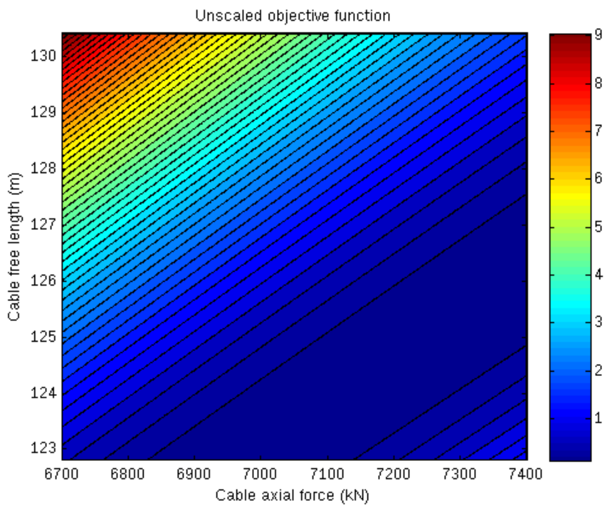

Figure 8 – Objective function

(I = m⁴)

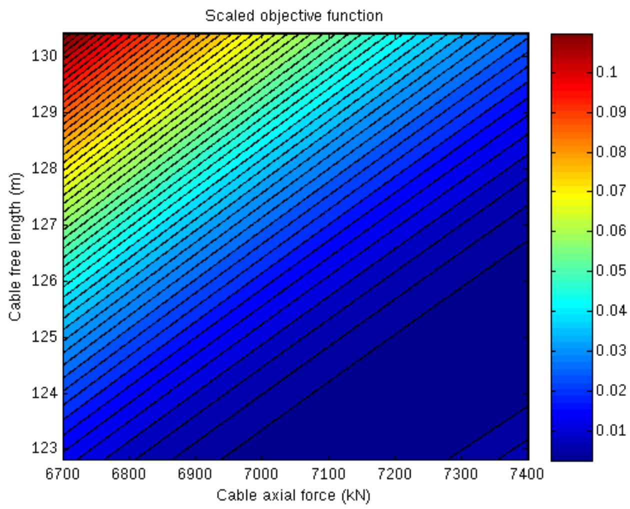

Figure 9 – Objective function

(I = m⁴)

Analysing the plots in Figures 8 and 9 one can see that, for a given bending stiffness of the cable, there are several possible combinations (N, L) that minimize the objective function. This conclusion is valid for both unscaled and scaled version of the objective function, although one can see that the use of the scaled version yields slightly higher values for the cable tensile force. The inexistence of a localized minimum for the objective functions means that the optimization problem cannot be run simultaneously for *L* and *N* and therefore, an uncertainty in *L* must be contemplated in the analysis. Once the cable free length is fixed, situation illustrated in Figures 10 and 11, one can see that the objective functions present a global minimum and therefore, the optimization problem can find a solution (*N*, *I*) that minimizes the differences between the experimental and theoretical natural frequencies of the cable. One can also see that the scaled version of the objective function yields, once again, slightly higher values for the cable tensile force.

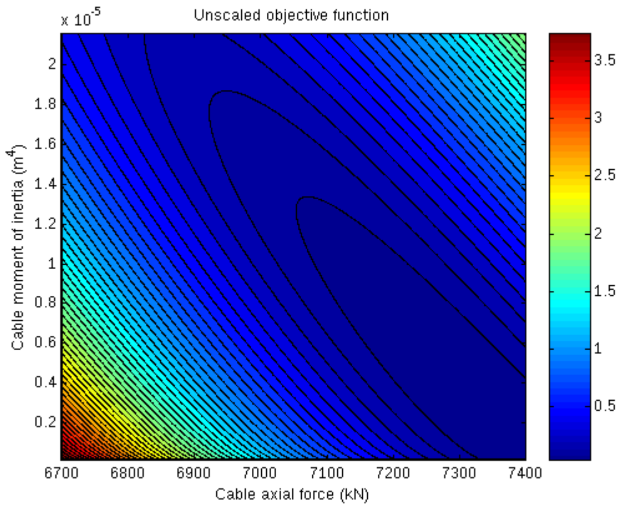

Figure 10 – Objective function (L = 124 m)

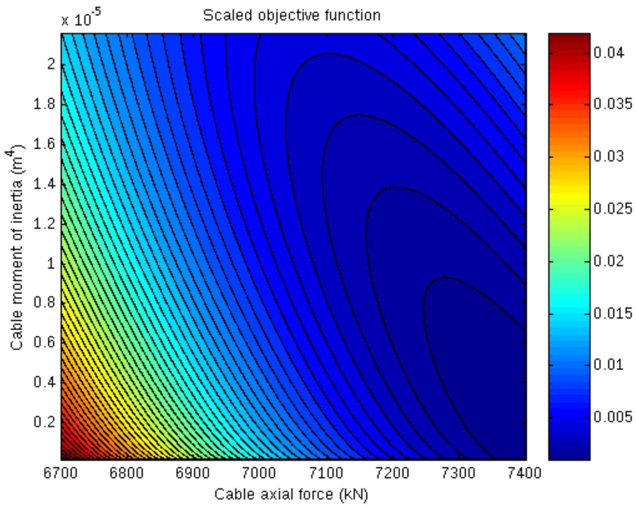

Figure 11 – Objective function (L = 124 m)

Based on these observations, the optimization problem was run for the unscaled and scaled version of the objective function defined by equations (13) and (14), for two cable free length of vibration, and , with and an initial guess for *N* (close to the design value). Figures 12 to 15 illustrate the optimization process for the T18t stay-cable.

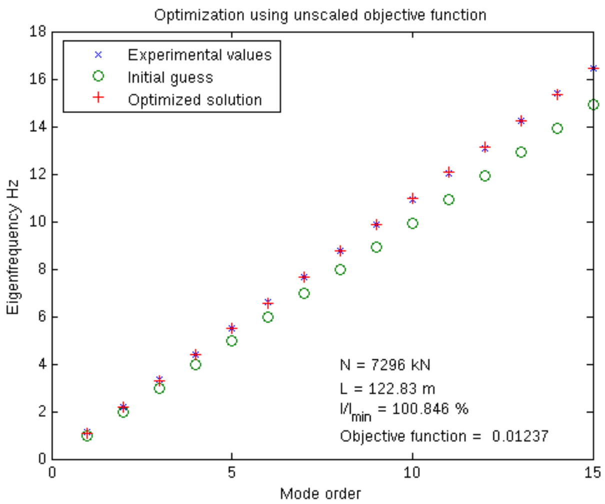

Figure 12 – Objective function ( )

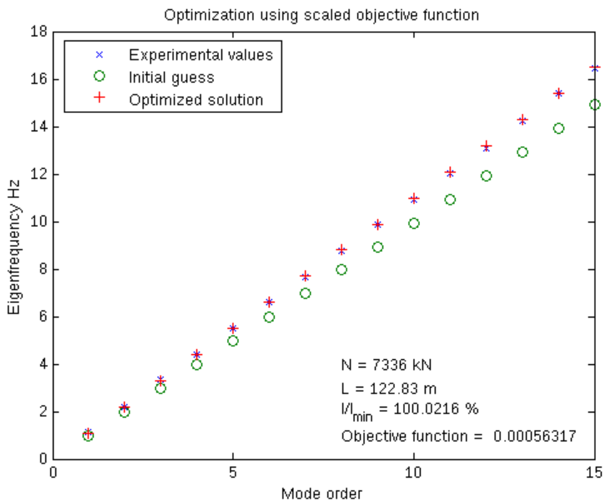

Figure 13 – Objective function ( )

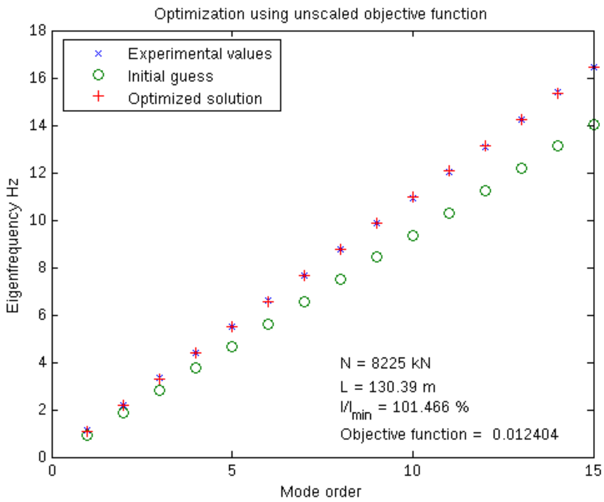

Figure 14 – Objective function ( )

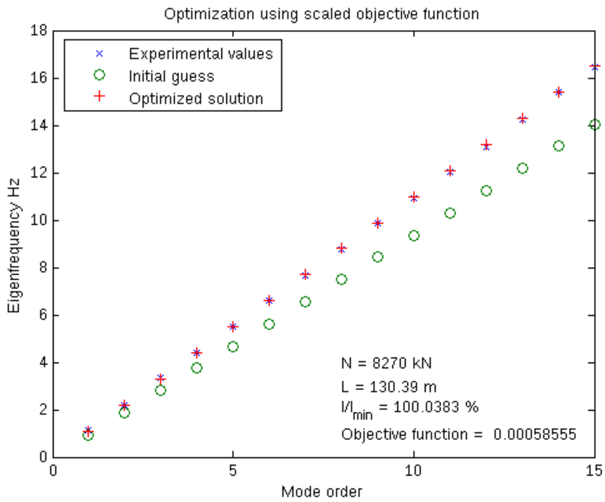

Figure 15 - Objective function ( )

Analysing the plots in Figures 12 to 15 one can see that, according to the applied methodology, the tensile force installed in T18t cable ranges between 7296 and 8270 kN. Therefore, one can estimate an average value of 7783 kN with an uncertainty of ±6%. This result is greatly influenced by the uncertainty in the definition of the cable free length of vibration since the solutions produced by the use of the scaled or unscaled version of the objective function are not significant. It is also possible to notice that the estimated bending stiffness of the stay-cable is very close to the value obtained considering , with a maximum deviation of 1.5%.

Table 1 and Figure 16 resume the optimization results for all the 18 stay-cables object of the present study. Figure 16 also shows the forces obtained from readings of the load-cells installed in some of the stay-cables of the bridge, which are a part of its monitoring system. The load-cells yield the force installed in one monostrand and the overall force in the corresponding stay-cable is extrapolated from this single reading, considering that the forces in all the monostrands are similar. These readings were performed in 2000 and 2010.

Table 1 - Optimization results (nif: number of identified frequencies)

Stay cable	nif	m (kg/m)	I min (m⁴)	I max (m⁴)	L min (m)	L max (m)	L min				L max				N estimation (kN)	ε (%)
							Unscaled objective function		Scaled objective function		Unscaled objective function		Scaled objective function			
							N (kN)	I/I_min (%)	N (kN)	I/I_min (%)	N (kN)	I/I_min (%)	N (kN)	I/I_min (%)		
T01t	4	100,901	1,48E-07	2,16E-05	25,32	30,75	4340	100,0	4528	100,0	6432	100,0	6709	100,0	5502	±18,0
T02t	5	75,374	1,11E-07	1,22E-05	30,96	35,87	4394	100,0	4503	100,0	5914	100,0	6060	100,0	5218	±13,9
T03t	6				36,06	41,16	4347	100,0	4438	100,0	5676	100,0	5794	100,0	5064	±12,6
T04t	6				41,44	46,70	4491	100,0	4521	100,0	5713	100,0	5751	100,0	5119	±11,0
T05t	7				46,80	52,31	4719	3192,7	4964	100,0	5896	4972,3	6210	100,0	5447	±12,3
T06t	7				52,51	58,14	5215	100,0	5265	100,0	6400	100,0	6462	100,0	5836	±9,7
T07t	8				58,11	63,96	5522	100,0	5566	100,0	6695	100,0	6749	100,0	6133	±9,1
T08t	13	84,150	1,24E-07	1,50E-05	63,87	69,93	5876	100,0	5910	100,0	7050	100,0	7091	100,0	6482	±8,6
T09t	9				69,67	75,91	5707	100,0	5775	100,0	6780	100,0	6861	100,0	6281	±8,5
T10t	13				75,51	81,92	6361	726,4	6431	100,0	7486	1016,0	7574	100,0	6963	±8,1
T11t	11				81,35	87,92	6721	449,3	6786	100,0	7851	605,9	7930	100,0	7322	±7,7
T12t	10	100,901	1,48E-07	2,16E-05	87,10	93,93	6797	100,0	6847	100,0	7908	100,0	7967	100,0	7380	±7,4
T13t	10				92,98	99,96	6990	100,0	7011	100,0	8082	100,0	8107	100,0	7548	±6,9
T14t	11				98,90	106,00	7160	100,0	7225	100,0	8228	100,0	8304	100,0	7729	±6,9
T15t	10				104,83	112,06	7045	100,0	7080	100,0	8054	100,0	8094	100,0	7568	±6,5
T16t	11				110,88	118,23	7052	100,0	7099	100,0	8021	100,0	8074	100,0	7562	±6,3
T17t	14				116,84	124,30	7182	100,0	7236	100,0	8131	100,0	8193	100,0	7686	±6,2
T18t	15				122,83	130,39	7296	100,9	7336	100,0	8225	101,5	8270	100,0	7782	±5,9

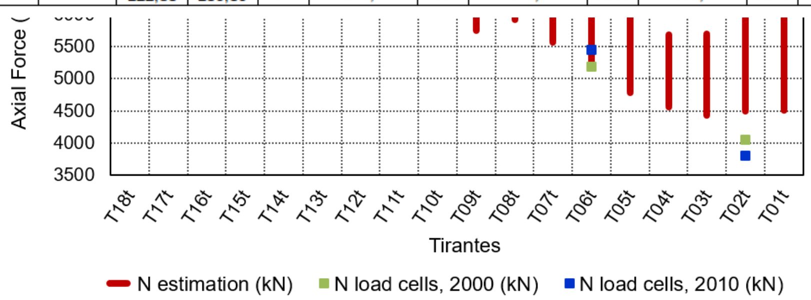

Figure 16 – Optimization results

The overall results for the force estimations show that the uncertainty associated with the force decreases as the length of the stay-cable increases. This is due to the fact that, as the stay-cables become longer, the ratio between and becomes closer to the unit, and hence the indefinition of the free length of vibration of the cables associated with the placement of the deviators becomes less important. Also AVT is less effective in shorter stay-cables, since they are more difficult to be excited by ambient vibration, leading to a fewer number of identified frequencies, as it can be observed in Table 1. This makes shorter stay-cables more difficult to characterize in terms of force since, due to their higher bending stiffness, they would need, as previously shown, a higher number of identified frequencies in order to yield good force estimations.

Although the readings provided by the load-cells give important information regarding the evolution of the forces in the stay-cables, the value of the forces themselves are subjected to a certain degree of uncertainty, due to the extrapolation procedure which is used in their definition. Unfortunately, it was also not possible to obtain up-to-date readings of the load cells, directly comparable with the estimations resulting from the proposed optimization procedure. In any case, the force estimations provided by the optimization process are consistent with the available readings from the load cells, although the estimations for stay-cable T02t seem too high.

CONCLUSIONS

The present paper presents a possible definition for a multi-variable optimization problem, used to simultaneously estimate the combination of cable tensile force, free length and bending stiffness that best match the cable measured modal properties. The application of this methodology is exemplified on eighteen stay-cables of Salgueiro Maia Bridge and from the performed analysis one can draw the following conclusions:

i) it was proven the inexistence of a localized minimum for the objective functions in terms of *N* and *L*, showing that the optimization problem cannot be run simultaneously for these varibles and therefore, an uncertainty in *L* must be contemplated in the analysis.

ii) once the cable free length is fixed the objective functions present a global minimum and the optimization problem can find a solution (*N*, *I*) that minimizes the differences between the experimental and theoretical natural frequencies of the cable.

Iii) the overall results for the force estimations show that the uncertainty associated with the force decreases as the length of the stay-cable increases.

iv) shorter stay-cables are more difficult to characterize in terms of force since, due to their higher bending stiffness, they need a higher number of identified frequencies in order to yield good force estimations. Unfortunately AVT provides fewer identified frequencies for these type of stay-cables.

v) in general, the force estimations provided by the optimization process are consistent with the available readings from load cells directly deployed in the stay-cable.

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REFERENCES

[1] Geier, R., De Roeck, G. and Petz, J., 2005, "Cable Force Determination for the Danube Channel Bridge in Vienna", *Structural Engineering International*, 15(3), 181-185.

[2] Geier, R., De Roeck, G. and Flesch, R., 2006, "Accurate cable force determination using ambient vibration measurements", *Structure and Infrastructure Engineering*, 2(1), 43-52.

[3] Caetano, E., 2011, "On the Identification of Cable Force from Vibration Measurements, *Proceedings IABSE-IASS 2011: Taller, Longer, Lighter,* 1-9.

[4] Morse, P. and Ingard, K., 1968, *Theoretical Acoustics*, International Series in Pure and Applied Physics, McGraw-Hill Inc., 911 pages.

[5] Mars, Ph. and Hardy, D., 1985, "Mesure des Efforts dans les Structures a Cables", *Annales TP Belgique*. vol. 6, 515-531.

[6] Robert, J., Bruhat D. and Gervais J., 1991, "Mesure de la Tension des Câbles par Méthode Vibratoire, *Laboratoire des Ponts et Chaussées.* vol. 173, 109-114.

[7] Mehrabi, B. and Tabatabai, H., 1998, "Unified Finite Difference Formulation for Free Vibration of Cables", *Journal of Structural Engineering.* vol. 124, 1313-1322.

[8] Ernest, H., 1965, "Der E-modul von Seilen unter Berücksichtigung des Durchhanges", *Bauingenieur.* vol. 40, 52-55.

[9] Irvine, H., 1981, *Cable Structures*, The Massachusetts Institute of Technology, 259 pages.

[10] Lanteigne, J., 1985, "Theoretical Estimation of the Response of Helically Armored Cables to Tension, Torsion, and Bending", *Journal of Applied Mechanics.* vol. 52, 423-432.