EWGLA

EURO Working Group on Locational Analysis

On risk management for a two-stage stochastic mixed 0-1 model for the design and operation planning of a closed-loop supply chain

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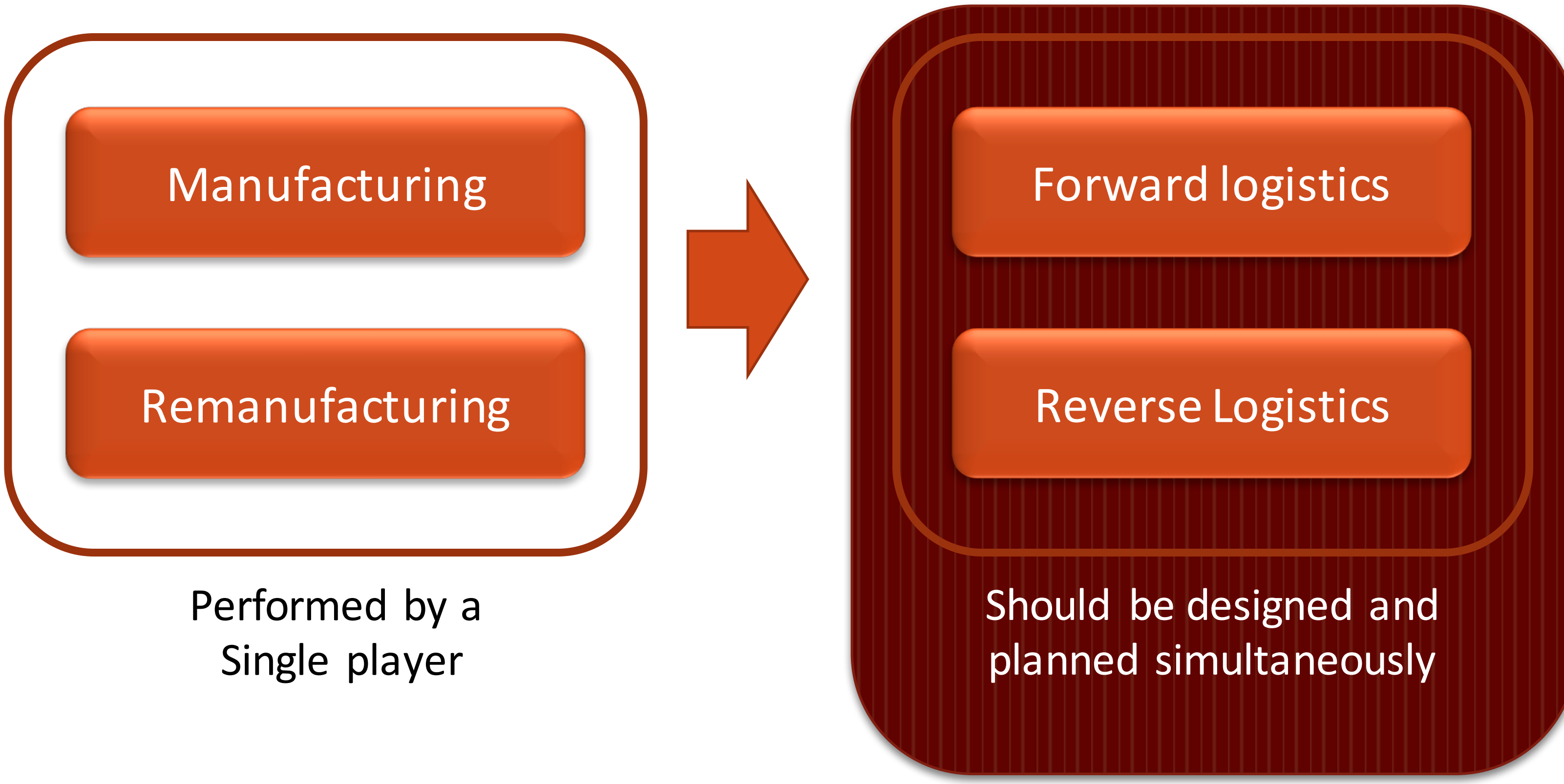
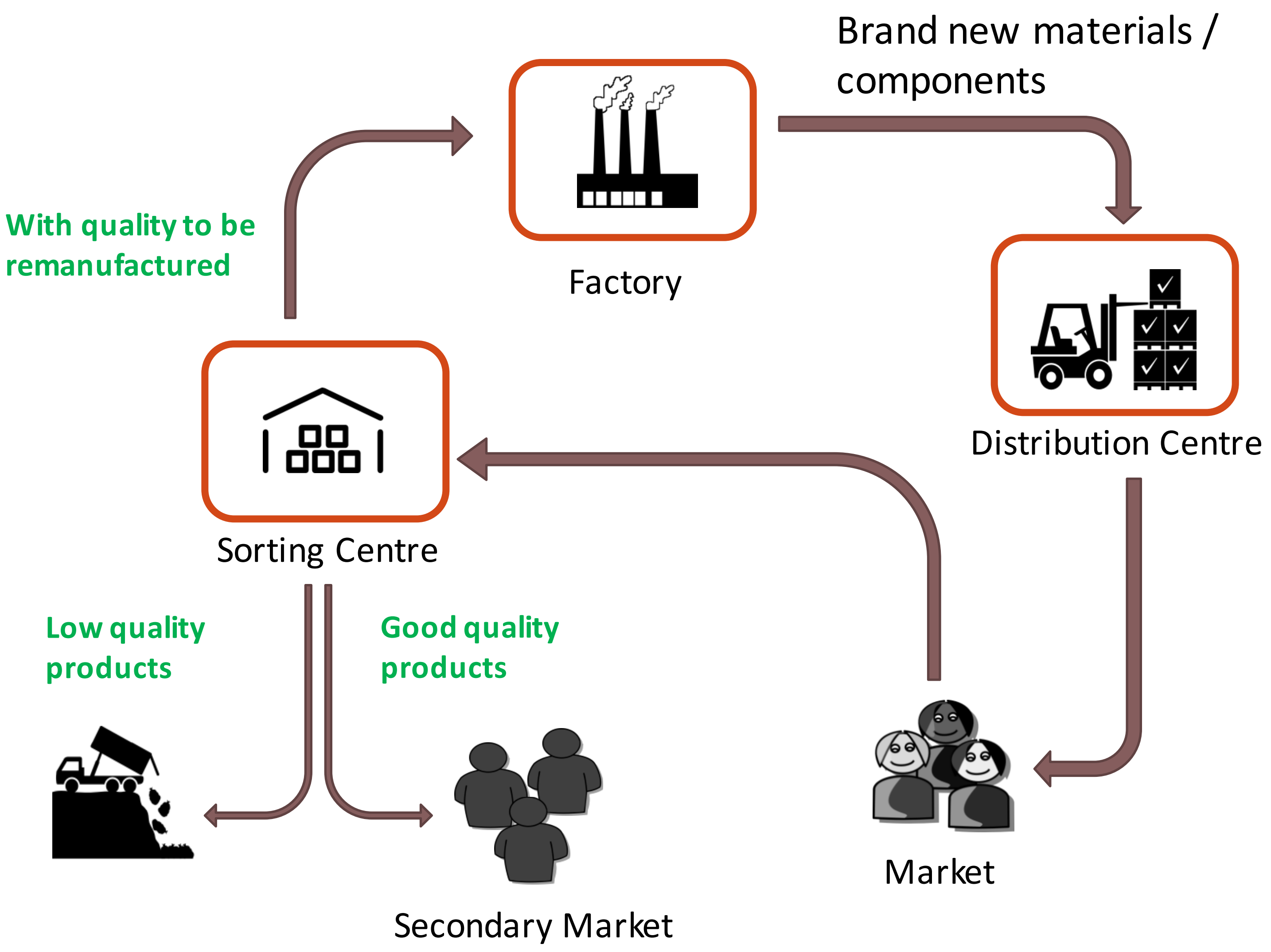
Motivation



CLOSED LOOP SUPPLY CHAIN

Supply chain structure

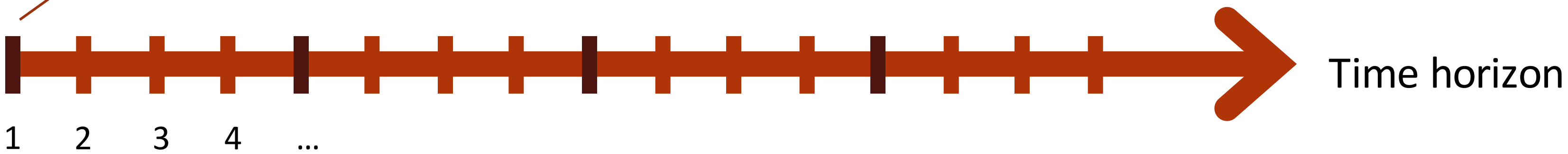


Supply chain decisions static

Time modeling

Long time horizon (e.g. 15 years)

Topological (strategic) decisions taken at the beginning of the time horizon (e.g. first-year period)



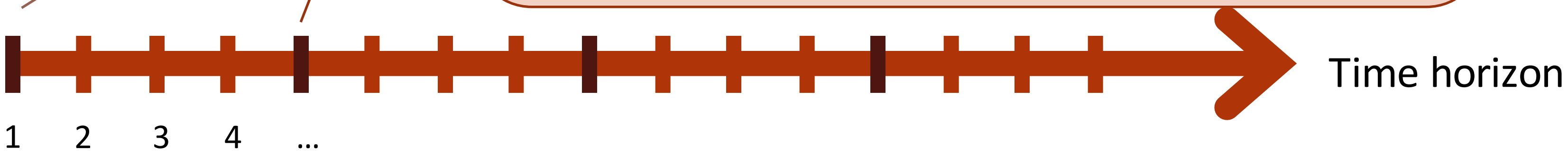
Operation planning decisions taken at smaller time scale (e.g. a year)

Supply chain decisions dynamic

Time modeling

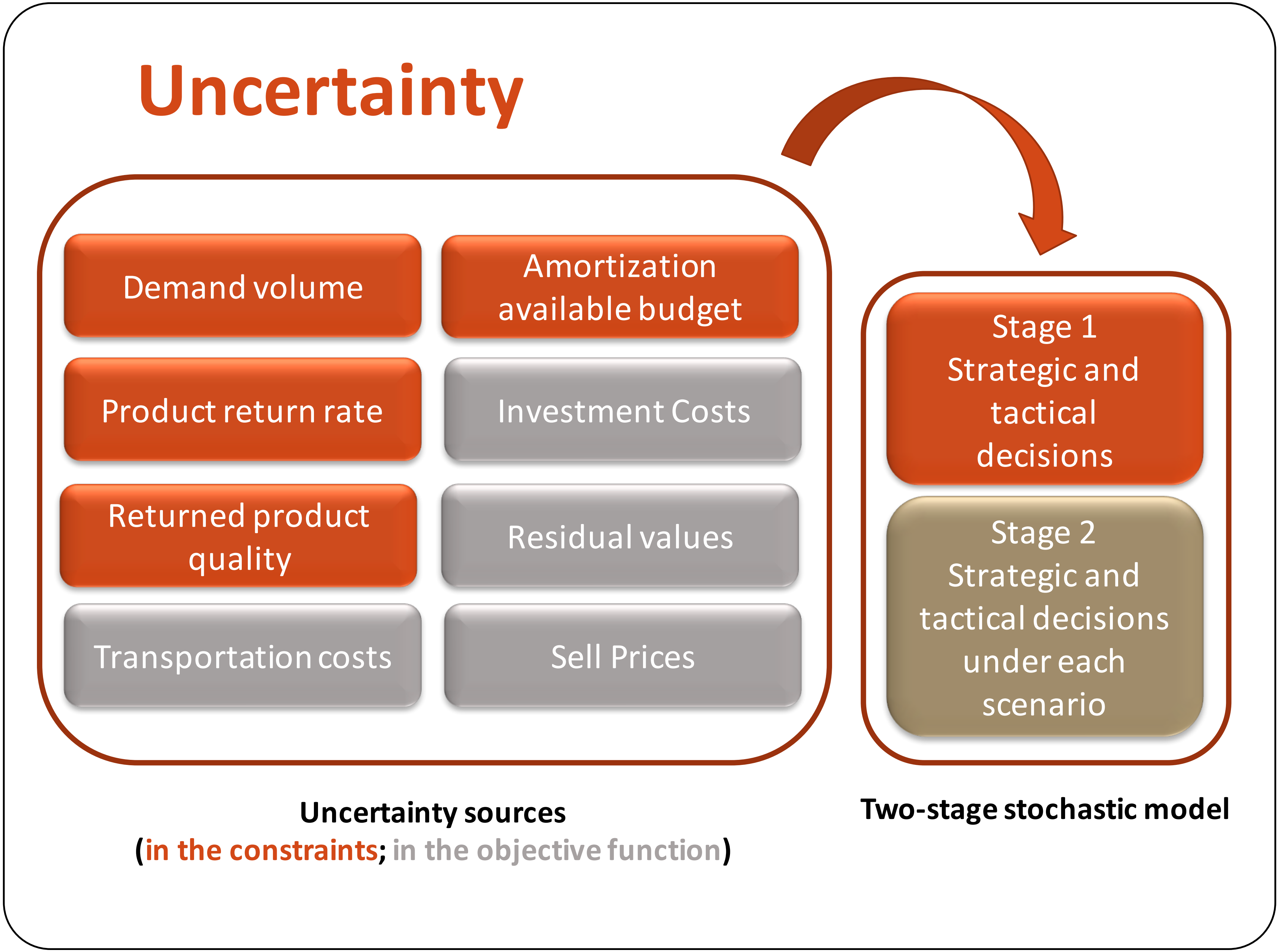
Topological (strategic) decisions

Macro periods: large time scale moments
(e.g. periods 1, 6 , 11)



Micro periods: smaller time scale
(e.g. all year periods 1 -15)

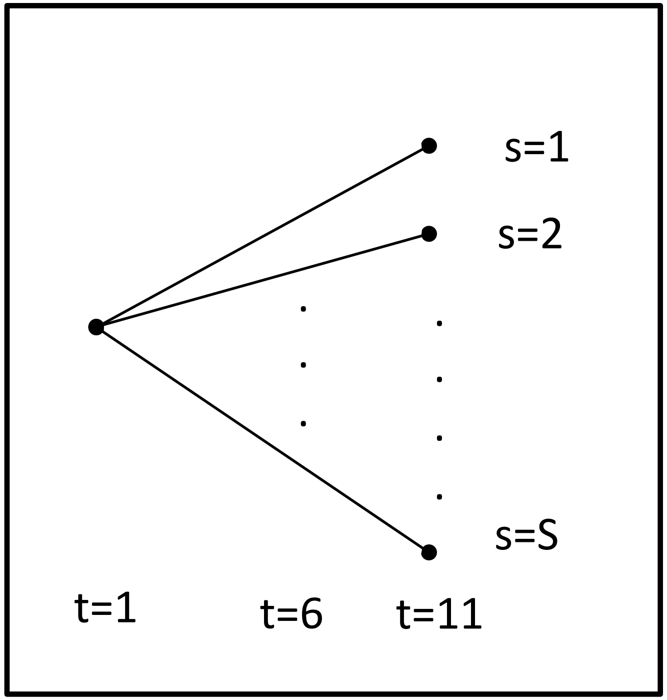
Tactical decisions



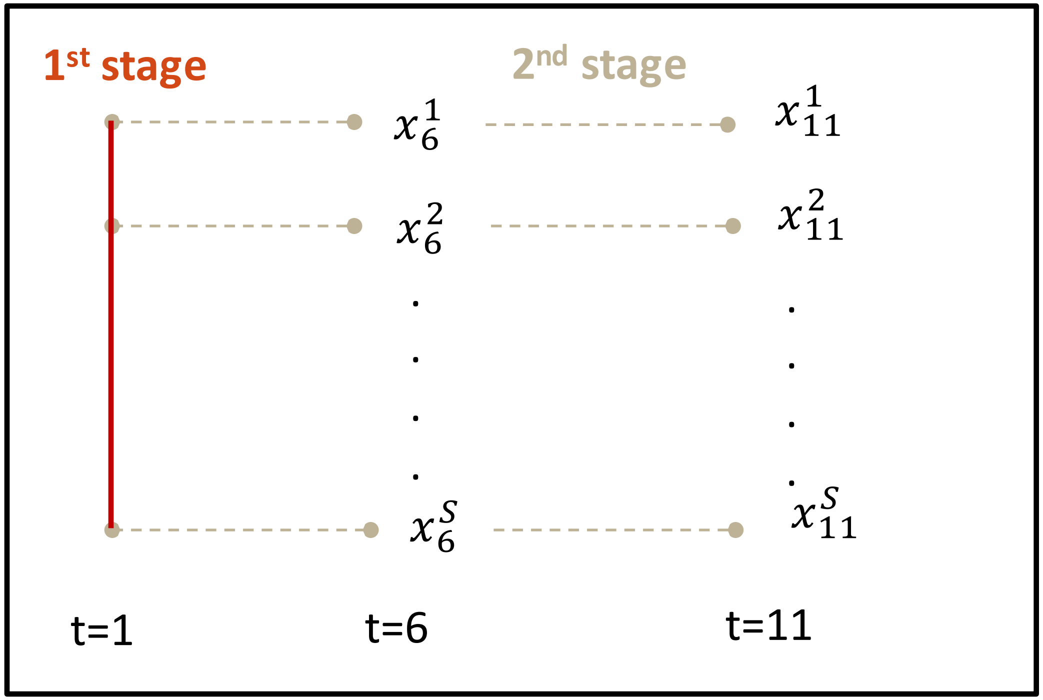
Uncertainty
Demand volume
Amortization available budget
Product return rate
Investment Costs
Returned product quality
Residual values
Transportation costs
Sell Prices
Stage 1 Strategic and tactical decisions
Stage 2 Strategic and tactical decisions under each scenario
Uncertainty sources
(in the constraints; in the objective function)
Two-stage stochastic model

Two-stage stochastic model

Data



Decisions



- Strategic and tactical decisions are single scenario based
- NAC are relaxed for all periods beyond the first one

Two-stage stochastic model

MACRO PERIODS

Factories locations and capacities

Distribution and sorting centers locations

1st stage (first macro period)

Distribution and sorting centers locations

2nd stage (all macro periods other than the first one)

Scenario dependent

MICRO PERIODS

Tactical planning

1st stage (first period) and

2nd stage (all periods beyond the first one) **decisions**

Scenario dependent

Location Decision Variables

δ_{ik} = 1 if factory *i* is operated with kth capacity (at macro period 1)

ξ_{i1}^{0} = 1 if a distribution or sorting centre *i* is contracted at macro-period 1

$\xi_{it'}^{\omega}$ = 1 if a distribution or sorting centre is contracted and made available at macro-period t' **(t'>1) for scenario** ω

New Location Decision Variables

Interactions between entities can only happen if both entities are available at that period

0-1 bilinear scheme modeling

$$MinF_{ij}\times\Phi_{it}^{\omega}\times\Phi_{jt}^{\omega}\leq X_{pijt}^{\omega}\leq MaxF_{ij}\times\Phi_{it}^{\omega}\times\Phi_{jt}^{\omega}$$

Bilinear terms replaced by

new 0-1 variable and new constraints*

$$\gamma_{ijt}^{\omega}\leq\phi_{it}^{\omega},\gamma_{ijt}^{\omega}\leq\phi_{jt}^{\omega},\phi_{it}^{\omega}+\phi_{jt}^{\omega}\leq 1+\gamma_{ijt}^{\omega}$$

where integrality in binary variable γ_{ijt}^{ω} can be relaxed

(*Fortet inequalities, RAIRO, 1960)

Risk Neutral Model

MAX net present value of the expected profit along the time horizon over the scenarios.

Subject to:

- Material balance equations for Factories and Distribution center, and for Sorting Centers regarding GOOD, REMANUFACTURING and POOR quality products
- Demand satisfaction
- Material balance equations for customers
- Factory capacity constraint: only one capacity can be selected
- Bounding the total cost annualized amortization of investments
- Upper bound and conditional lower bound on production at factories and on product flow between entities
- Stock upper bounding at the entities

Risk Neutral Model

Traditional approach to uncertainty allows to introduce in the model scenarios that represent the uncertainty however it provides a solution that **ignores the variability** of the objective function value over the scenarios (if any)

It does not minimize (or at least, reduce) the impact of bad scenarios (the one with **low-probability** but **high-bad** consequence)

In our case, the “left” tail of non-wanted scenarios

Time Stochastic Dominance (TSD) strategy

Consider a set of user-driven risk profiles, each defining at given macro-periods, the 4-tupla :

- **profit threshold**
- a bound target on the **probability of failure due to a profit shortfall** (First Stochastic Dominance)
- a bound target on the **expected profit shortfall** (Second Stochastic Dominance)
- **a bound on the maximum profit shortfall**

Gollmer, R., Neise, F., Schultz, R. (2008). SIAM Journal on Optimization, 19:552 571.
Gollmer, R., Gotzes, U., Schultz, R.,(2011). Mathematical Programming, Ser. B, 126:179 190.

Time Stochastic Dominance Risk Averse Model

$$
\begin{aligned}
z_{SDC} = \quad & \max\ a_1 x_1 + b_1 y_1 + \sum_{\omega \in \Omega} w^\omega \Big(\sum_{t \in \mathcal{T}_m^-} a_t^\omega x_t^\omega + b_t^\omega y_t^\omega \Big) - \sum_{t' \in \mathcal{T}^*} \sum_{p \in \mathcal{P}_{t'}} (M_e^p \epsilon_e^p + M_\beta^p \epsilon_\beta^p) \\
\text{s.t.} \quad & A_1 x_1 + B_1 y_1 = h_1 \\
& A_t'^\omega x_1 + B_t'^\omega y_1 + A_t^\omega x_t^\omega + B_t^\omega y_t^\omega = h_t^\omega \\
& a_1 x_1 + b_1 y_1 + \sum_{t \in \mathcal{T}: 1 < t < t'} (a_t^\omega x_t^\omega + b_t^\omega y_t^\omega) + s_{t'}^{\omega p} \geq \phi_{t'}^p \\
& \sum_{\omega \in \Omega} w^\omega s_{t'}^{\omega p} \leq e^p + \epsilon_e^p \\
& 0 \leq s_{t'}^{\omega p} \leq S_{t'}^p \nu_{t'}^{\omega p} \\
& \sum_{\omega \in \Omega} w^\omega \nu_{t'}^{\omega p} \leq \beta^p + \epsilon_\beta^p \\
& x_1 \in \{0,1\}^{nx_1},\ y_1 \in I\!R^{+ny_1} \\
& x_t^\omega \in \{0,1\}^{nx_t^\omega},\ y_t^\omega \in I\!R^{+ny_t^\omega} \\
& 0 \leq \epsilon_e^p \leq S^p - e^p,\ 0 \leq \epsilon_\beta^p \leq 1 - \beta^p \\
& \nu_{t'}^{\omega p} \in \{0,1\}
\end{aligned}
$$

Penalization for bounds violation

Shortfall variable definition

Expected shortfall bound target

Shortfall maximum value

Shortfall probability bound target

Time Stochastic Dominance Risk Averse Model

Expected shortfall bound target

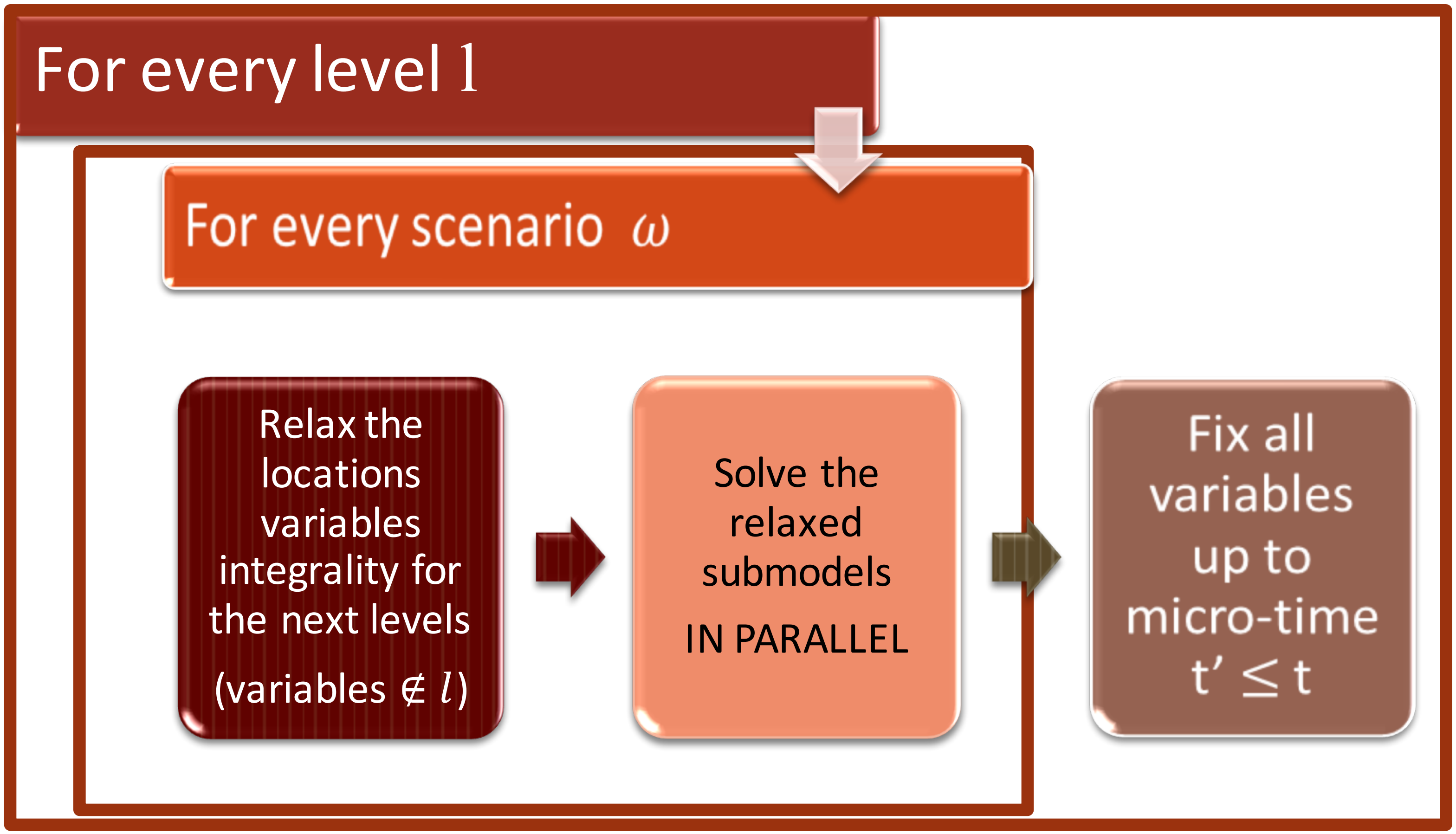
$$\sum_{\omega\in\Omega} w^{\omega} S_{t\prime}^{\omega p} \leq e^{p} + \epsilon_{e}^{p} \quad \Leftarrow \quad w^{\omega} S_{t\prime}^{\omega p} \leq e^{p} + \epsilon_{e}^{p} \quad \forall\omega \in \Omega$$

Shortfall probability bound target

$$\sum_{\omega\in\Omega} w^{\omega} \vartheta_{t\prime}^{\omega p} \leq \beta^{p} + \epsilon_{\beta}^{p} \quad \Leftarrow \quad w^{\omega} \vartheta_{t\prime}^{\omega p} \leq \frac{\beta^{p}}{n^{p}} + \epsilon_{\beta}^{p} \quad \forall\omega \in \Omega$$

where n^p is defined maximum number of scenarios with shortfall in the profile p

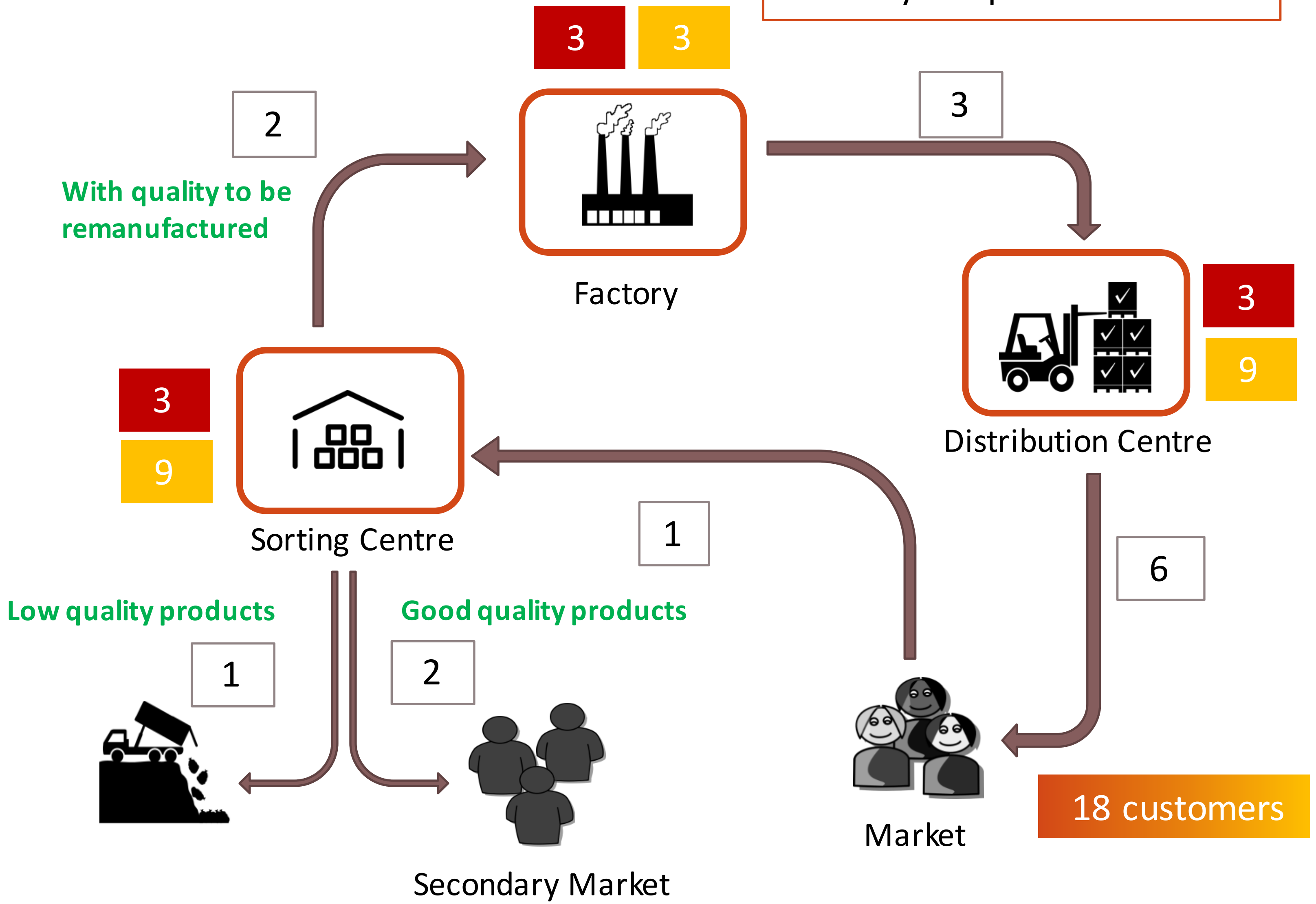
Heuristic


For every level l
For every scenario ω
Relax the locations variables integrality for the next levels
(variables $\notin l$)
Solve the relaxed submodels
IN PARALLEL
Fix all variables up to micro-time t' ≤ t

Pilot cases

Case 1 | Case 2 | Products

Time scale: 15 micro-periods
3 macro-periods
3 factory's capacities



Instances’ dimensions

	Scen	Constraints	Binary var	Contin. var	Density (%)
C1	12	193 165	159	87 894	0.0039
C2	12	522 196	495	253 308	0.0014

	Scen 1	Scen 2	Scen 3	Scen 4	Scen 5	Scen 6	Scen 7	Scen 8	Scen 9	Scen 10	Scen 11	Scen 12
P1	0.01	0.01	0.03	0.1	0.1	0.1	0.15	0.25	0.15	0.05	0.04	0.01
P2	0.01	0.01	0.01	0.02	0.05	0.1	0.05	0.15	0.10	0.18	0.12	0.20

Heuristic results

CPLEX *vs* Fix and Relax Algorithm

	z_{CPLEX} (10^6)	z_{alg} (10^6)	GAP_z %	t_{CPLEX}	$t_{par\downarrow alg}$
C1P1	-10.69*	-10.6909	0.003	7 426	1 165
C1P2	-11.26*	-11.2588	0.03	10 511	948
C2P1	—**	-120.432	-	28 800	19 157
C2P2	- 130.968**	-120.678	-	28 800	20 522

* optimal solution

** best solution obtained within the time limit 28 800 s

Note: negative values are due intentionally to high penalties of risk averse bound targets

Technical information: WS with a 2 Intel Xeon E5430 266 GHz processor (4 cores each), 24 GB of RAM *gcc* 4.9.2 as C++ compiler and CPLEX 12.6 as MIP engine

Heuristic results

risk neutral *vs* risk averse

In 10^6	$y^{RN}{}_{CPLEX}$	$y^{RA}{}_{alg}$
C1P1	5.55307	5.55308
C1P2	4.98132	4.98104
C2P1	$-^{**}$ (3.810)	3.736
C2P2	0.8243^{**} (3.631)	3.544

** best solution obtained within the time limit 28 800 s
(…): best upper bound value given by CPLEX

$y^{RN}{}_{CPLEX}$: expected profit solution value of the Risk Neutral model computed by CPLEX plain use

$y^{RA}{}_{alg}$: expected profit solution value of the Risk Averse model computed by the heuristic (without penalty terms)

Final remarks and future work

Novelty

- **Topological decisions** are now **dynamic** decisions to be taken at different periods of the time horizon
- **Simultaneous availability** of two entities at a given period is now considered by 0-1 bilinear terms replaced by linear ones
- **Several sources of uncertainty**
  - Product: demand, sell prices, transportation costs, return rates, returned products quality
  - Financial: amortization available budget, investment costs and residual values
- **New TSD heuristic**
  - Parallelize computational implementation

In the near future

- Refine the upper bound scheme provider for getting stronger bounds

References

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