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A Two-Stage Stochastic Model for the Design and Planning of a Multi-Product Closed Loop Supply Chains

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Abstract

In this paper we address the problem of uncertainty in the design and planning of a multi-period, multi-product closed loop supply chain, where the recovered products are end-of-life products that are disassembled and recycled. Uncertainty is explicitly modelled by considering customers' demands and returns to be stochastic. A two–stage model is developed where first stage decisions concern the facility location while second stage decisions are the production planning of the supply chain. The integer L-shaped method was adopted as the solution tool and computational tests were performed on multi-period and multi-commodity networks randomly generated based on a reference case. A comparison between the proposed solution method and the straight use of the CPLEX is performed.

Keywords: Closed-Loop Supply Chain, Design and Planning, Two-stage Stochastic Optimization

1. Introduction

The research in closed loop supply chains (CLSC) has significantly increased in recent years driven by an increased concern of society towards the minimization of resources usage. This has imposed the creation of new legislation essentially in Europe and United States of America that implied a change at the companies' level when dealing with their supply chains leading to the appearance if the Closed loop supply chains. These go beyond the classical forward supply chains challenges since typically more players are involved (reverse logistics or product disposition may be provided by contracted third-parties). Also in such structures players interests may be in conflict (durable components are attractive for the manufacturer due to remanufacturing but for the supplier this durability means a sales loss) and new relationships exist between the product collection rate, the product durability and the product life cycle (Guide and Wassenhove, 2009). However, if the manufacturing and remanufacturing systems are operated by the same player, as it happens in some of the electronic and automobile industries, important benefits may be obtained by considering simultaneously strategic decisions such as the facilities number and their location's and tactical decisions such as production, inventory and distribution planning.

One of the main problems existing when dealing with CLSC networks is, apart from the common uncertainty in the demand, the uncertainty in the availability, namely timing, quantity and quality of used products. Few network design models for CLSC have however addressed this uncertainty explicitly. Stochastic demands and returns have been considered in the context of a single product network design by Listes (2007) and Inderfurth (2005) for a single period and multi-period respectively, while models developed by Salema et al. (2007) and Chouinard et al. (2008) focused on static (single period) multi-product networks. You et al. (2009) addressed uncertainty in simple forward supply chains as well as did more recently Georgiadis et al. (2011).

In this paper we will focus on the design and planning of a multi-period, multi-product closed loop supply chain where end-of-life products are collected, disassembled and recycled. Uncertainty is explicitly considered in the model by assuming customers' demands and returns to be stochastic. The model here proposed is a two-stage stochastic model that extends the general modeling framework developed by Salema et al. (2010). Strategic decisions involving the four echelon network facilities (plants, warehouses, customers and disassembly centers) are made in the first stage, while the acquisition, production and logistics planning are decided in the second stage. We assumed that the random vector ξ has finite support and s=1,…,n indexes the possible realizations (scenarios) of ξ, being p_s their probabilities. As in the deterministic model two time scales were considered: a macro scale for strategic decisions time and a micro scale for planning decisions. The integer L-shaped method is adopted as the solution tool and computational tests are performed on multi-period and multi-commodity networks randomly generated from a reference case previously addressed by the authors (Salema et al, 2010).

2. Problem formulation

The problem is formulated as a two-stage stochastic model so that strategic decisions involving the location of the four echelon network facilities (plants, warehouses, customers and disassembly centers) are made in the first stage, while each scenario's production and logistics planning are decided in the second stage.

Briefly, the model defines the entity *i* (plant, warehouses, customers or disassembly centre) to be opened /served, and for each scenario *s*, the amount of product *m* served by entity *i* to entity *j* at micro-time period *t*, the amount of product *m* stocked in *i* at micro-time *t*, and the customers' unmet demand at macro-period T, so that the expected total supply chain cost is minimized. The objective function involves a deterministic cost, the first stage cost, and an expected second stage cost that is equal to the product of the scenario probability p_s by the associated second stage scenario cost, summed over all scenarios. The first stage cost is composed by the entities' opening/use fixed costs and the penalty costs for leaving a customer out of the supply chain, while the second stage cost under each scenario *s* is composed by the cost of the lost demand of all customers for all products in all macro-time periods, the inventory costs for all products at all entities (with the exception of customers) for all micro-time periods, and flow costs for all products between all entities for all micro-time periods. Plants' maximum and minimum production capacities, maximum and minimum flows' capacities, and mass balance equations for all entities are ensured for all scenarios. Briefly the two-stage problem obtained follows the general form:

$$\min c_o{}^T x + p_1 c_1{}^T y_1 + p_2 c_2{}^T y_2 + \cdots + p_n c_n{}^T y_n$$

$$\text{s. t.} \quad \begin{array}{llllll} T_1 x + & W_1 y_1 & & & = & h_1 \\ T_2 x + & & W_2 y_2 & & = & h_2 \\ \vdots & & \dots & & = & \vdots \\ T_n x + & & & W_n y_n & = & h_n \end{array}$$

$$x \in B, y_1 \geq 0, y_2 \geq 0, \dots, y_n \geq 0$$

where x denotes the vector of the first stage variables from the B set i.e. the hypercube with the appropriate dimension, y_s the second stage variables under scenario s and c_o is the deterministic first stage cost.

3. Solution methods

From the general form given above it is clear that the problem is a large mixed integer linear problem that presents a special structure: the dual angular structure. A classical approach to such problems is the Bender's decomposition that iteratively solves the n linear subproblems in the variables y_s where the coupling variables x are fixed to a given value. The optimal simplex multipliers thus obtained (π_s) generate a new cut

$$\theta \geq \sum_{s=1}^{n} p_s (\pi_s)^T h_s - \left(\sum_{s=1}^{n} p_s (\pi_s)^T T_s \right) x$$

that is added to the first stage problem (master problem)

$$\begin{aligned} & \min c_o{}^T x + \theta \\ \text{s.t.} \quad & \theta \geq d_l - e_l x, \quad l = 1, \ldots \\ & x \in B \end{aligned}$$

The first stage optimal solution values set the values of the coupling variables for the next subproblems iteration in the variables y_s. The procedure is repeated until the θ value exceeds the subproblems expected cost function value. This procedure has been largely applied in the context of two-stage linear recourse programming with continuous first and second stage variables and was proposed by Van Slyke and Wets (1969) as the L-shaped method. Other than the cuts previously indicated and known as optimality cuts, the L-shaped method adds feasibility cuts to the master problem in order to ensure that the first stage solution is feasible for the second stage subproblems. In the multi-period, multi-product closed loop supply chain design and planning problem it is clear that every first stage solution *x* also has a feasible solution in the second stage, so that the stochastic program has the relatively complete recourse property. Therefore only optimality cuts were considered. The solution procedure adopted was the L-shaped extension to integer variables due to Laporte and Louveaux (1993) and known as the integer L-shaped method. In particular the optimality cuts are defined as:

$$\theta \geq (\theta_r - L)(\textstyle\sum_{i \in S_r} x_i - \sum_{i \notin S_r} x_i) - (\theta_r - L)(|S_r - 1| + L$$

where S_r is the r^{th} feasible solution, θ_r the corresponding expected second-stage value and L is a lower bound of the expected second stage cost. In this case we set the L value as the linear relaxation problem (LRP) expected second-stage cost and explored successively the N_0, and N_1 neighborhoods of the LRP solution until optimality was reached. Notice that the t-neighborhood of solution S_r is defined as the set of all solutions so that $\left|\sum_{i \in S_r} x_i - \sum_{i \notin S_r} x_i\right| = |S_r| - t$.

4. Results analysis

The computational tests here presented were performed on multi-period and multi-commodity networks randomly generated from a reference case previously addressed in Salema et al (2010). In particular for a 5-year time horizon, with a one-year macro-time unit and two-months micro-time unit, we randomly generated 5 instances of a network with 5 plants, 10 warehouses, 25 customers and 10 disassembly centers. Concerning the products, 3 different products were considered in the flows plants-warehouses, 6 for the flows warehouses-customers, 1 for the flows customers-disassembly centers and finally 2 for the closing loop flows. Regarding the number of scenarios, a pessimistic, expected and optimistic scenarios were identified. The customers' demands and returns expected values and standard deviations were set as $\mu_t + \alpha\sigma_t$ and $\mu_p + \alpha\sigma_p$, respectively, where μ_t and σ_t are denoted for the macro-period t and μ_p and σ_p are denoted for product p. The parameter α was set to the values -1, 0 and 1 according to the pessimistic, expected and optimistic scenario, respectively. In order to establish customers' demands expected values and standard deviations, product demands were generated according to a uniform distribution U[500,2000] and μ_t was defined as $\mu_t = \beta_t \times product\ demand$ and σ_t was set as $\frac{\sigma_t}{\mu_t} = \gamma_t$. The values of the parameters β_t and γ_t are given in the table below.

Table 1 – Customers' demands parameters

	t=1	t=2	t=3	t=4	t=5
β_t	0.5	0.52	0.55	0.48	0.49
γ_t	0.05	0.20	0.20	0.40	0.40

Finally, for the returns' expected values and standard deviations, μ_p was defined as $\mu_p = \delta_p \times product\ demand$ and σ_p was set as $\frac{\sigma_p}{\mu_p} = \varepsilon_p$. The values of the parameters δ_p and ε_p are shown below.

Table 2 – Returns' parameters

	p 1	p 2	p 3	p 4	p 5	p 6
δ_p	0.45	0.7	0.5	0.8	0.4	0.9
ε_p	0.20	0.10	0.20	0.05	0.20	0.05

The table below presents the computational times of CPLEX 12.2 and the integer L-shaped method in order to solve up to optimality the different problem instances. Both methods run on a laptop with a 2.4 GHz Core i5 processor. Notice that while CPLEX was straight used to solve the mixed integer problem, the L-shaped required the solution of the linear relaxation problem. Since this preprocessing was automatically integrated in the method, the computational times given below already include this aspect.
The cases studied involved 121050 equations and 267800 variables, being 50 discrete variables.

Table 3 – Computational Times

	CPLEX 12.2 (*seconds*)	Integer L-shaped (*seconds*)	Time reduction (%)
Network 1	2068	818	60
Network 2	1152	246	79
Network 3	1149	322	72
Network 4	1609	524	67
Network 5	4522	225	95

Regarding the results here presented, two major facts should be stressed. First, the integer L-shaped method solves up to optimality all problem instances and outperforms the commercial software CPLEX. The computing time reductions face to the CPLEX results range from 60 up to 95%. Second, though the number of decision variables involved in the different networks is the same, the computing performance exhibits a significant variability due to the network structure variability. Experiences involving larger networks were also carried out but the need of exploring the N_2 neighborhood of the LRP solution, led to unconcluded results.

5. Conclusions

In this work we proposed a two-stage stochastic model for the design and planning of closed-loop supply chains where customers demands' and returns' are uncertain. The integer L-shaped method was adopted to face the multi-period environment. The computational tests performed on a multi-period and multi-commodity networks randomly generated showed that the solution technique outperformed the commercial solver CPLEX.

As future work the authors aim to test the solution procedure in more complex networks with larger number of variables. In a first approach, tests will be conducted within the same context of the pessimistic, expected and optimistic scenarios. More scenarios are also to be considered, but since the solution of the linear relaxation problem will be at some point no longer possible, a different solution method will have to be adopted. The sample average approximation rises then as the appropriate solution tool to be analyzed.

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