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A Metaheuristic for Solving Large-Scale Two-Stage Stochastic Mixed 0-1 Programs with a Time Consistent Stochastic Dominance Constraints Risk Averse Strategy

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Abstract

In the present paper, we study general two-stage stochastic mixed 0-1 problems, in the context of closed loop systems where forward and reserve chains are simultaneously considered. Besides the fact that uncertainty appears anywhere in the objective function, right hand side and constraint matrix coefficients, the 0-1 variables and continuous variables have nonzero coefficients in both stages. A metaheuristic algorithm is proposed as a specialization for two-stage problems of the so named Fix-and-Relax Coordination Algorithm (FRCA) for solving large-scale multiperiod stochastic mixed 0-1 optimization problems under a time stochastic dominance constraint strategy, so-named TSD. This strategy is a mixture of the first and second-order stochastic dominance constraint risks averse measures induced by mixed-integer linear recourse that are considered in [Gollmer et al., 2011] plus enduring this strategy along the time horizon see [L.F. Escudero and Pérez, 2014]. In order to assess the solution procedure applicability, computational tests were performed on multi-period and multi-commodity closed loop supply chains randomly generated from a deterministic reference case previously addressed in [Salema et al., 2010]. Due to the large scale of the deterministic case involved, the stochastic counterparts addressed are very large ones.

Keywords: Two-stage stochastic mixed 0-1 optimization, metaheuristic, Fix and Relax, time stochastic dominance

1. Introduction

Supply Chains are complex networks of entities and flows that recently have evolved to consider closed loop systems where forward and reserve chains are simultaneously considered. Under this extended systems the treatment of uncertainty has been recognized as crucial although the resulting problems, usually large-scale two-stage stochastic problem result in complex system to be solved.

In the present paper we aim to contribute to the solution of this problem. We study general two-stage stochastic mixed 0-1 problems, where the uncertainty appears anywhere in the objective function, right and side and constraint matrix coefficients. The 0-1 variables and continuous variables have nonzero coefficients in both stages. A metaheuristic algorithm is proposed as a specialization for two-stage problems of the so named Fix-and-Relax Coordination Algorithm (FRCA) for solving large-scale multiperiod stochastic mixed 0-1 optimization problems. So, in the multiperiod multistage case as in the multiperiod two-stage case the large-scale character can be motivated by the intrinsic dimensions of the problem as well as due to the large number of additional 0-1 and continuous variables and constraints required by the risk averse strategy to use. In this work it is the time consistent stochastic dominance constraint strategy, so-named SDC, as mixture of the first and second-order stochastic dominance constraint risks averse measures induced by mixed-integer linear recourse that are considered in [Gollmer et al., 2011] plus considering a time consistency policy, see [L.F. Escudero and Pérez, 2014].

A main characteristic of the inexact approach that is proposed consists of considering so-named levels along the time horizon included by disjoint sets of consecutive periods. At each level an independent partial scenario mixed 0-1 model is solved by fixing the 0-1 variables of ancestor levels to the value obtained at the optimization of their related models and relaxing the integrality of the 0-1 variables related to successor levels as well as the (cross scenario SDC systems) for those other levels. Additionally, the continuous variables are obtained at each level by considering the bound targets of the SDC systems up to that level.

In order to assess the solution procedure applicability, computational tests were performed on multi-period and multi-commodity closed loop supply chains randomly generated from a deterministic reference case previously addressed in [Salema et al., 2010], that being a large scale case, its stochastic counterpart is a very large one.

The rest of the work is organized as follows. In Section 2 the two-stage mixed 0-1 optimization problem with the risk averse strategy TSD is presented, as well as the model for the risk neutral one. Section 3 presents our specialization of the the Fix-and-Relax Coordination (FRCA) to the two-stage problem with strategy TSD. Section 4 reports the main results of a computational experience for assessing the validity of the proposal made in this work. Section 5 concludes.

2. Two-stage stochastic mixed 0-1 problems under time stochastic dominance (TSD) strategy

It is well known that the risk neutral (expected) objective function maximization has the inconvenience of providing a solution that ignores the variability of the function value over the scenarios, if any. So, it does not hedge against the occurrence of low-probability high-bad consequence outlooks (i,e., the so named ”black swan” events).

Alternatively, there are some other approaches that also deal with risk management by providing hedging solutions against the occurrence of some non-desired scenarios considering risk averse strategies, see a survey in [Alonso-Ayuso et al., 2014]. Some of these strategies consider semi-deviations, excess probabilities, conditional value-at-risk, expect shortfall on reaching given thresholds as risk measure-based functions to optimize as well as the two recent new risk averse measures, namely, the first- and second-order Stochastic Dominance Constraint strategies. Those later ones are defined by a set of profiles included by thresholds on given profit values and some types of shortfall related bounds on reaching them.

In this paper we consider the specialization of a time stochastic dominance risk averse strategy for solving multistage problems introduced in [L.F. Escudero and Pérez, 2014] to the two-stage environment. That strategy so-named TSD is a time-based mixture of the FSD and SSD strategies. One of the main differences of the new strategy and the other ones are the bound on the probability of failure of reaching the thresholds, the expected shortfall on reaching them and the maximum shortfall are not hard constrains but targets to reach.

Without loss of generality, let the following compact representation of the two-stage stochastic mixed 0-1 DEM for maximizing the expected objective function value over the set of scenarios along a time horizon by only considering the risk neutral (RN) strategy,

$$
\begin{array}{lll}
z_{RN} = & \max\{a_1x_1 + b_1y_1 + \sum_{\omega\in\Omega} w^\omega \sum_{t\in\mathscr{T}^-} (a_t^\omega x_t^\omega + b_t^\omega y_t^\omega)\} & \\
\text{s.t.} & A_1x_1 + B_1y_1 = h_1 & \\
& A_1^{t\omega}x_1 + B_1^{t\omega}y_1 + \sum_{t'\in\mathscr{T}^-:t'\le t} (A_{t'}^{t\omega}x_{t'}^\omega + B_{t'}^{t\omega}y_{t'}^\omega) = h_t^\omega & \forall t\in\mathscr{T}^-,\ \omega\in\Omega \\
& x_1 \in \{0,1\}^{nx(1)},\ y_1 \in \mathbb{R}^{ny(1)} & \\
& x_t^\omega \in \{0,1\}^{nx(t)},\ y_t^\omega \in \mathbb{R}^{ny(t)} & \forall t\in\mathscr{T}^-,\ \omega\in\Omega,
\end{array}
\tag{1}
$$

where w^ω is the modeler-driven weight or likelihood of scenario ω; x_1 and y_1 are the vectors of the first stage 0-1 and continuous variables, respectively; a_1 and b_1 are the objective function coefficient vectors for the variables in the first stage vectors x_1 and y_1, respectively; A_1 and B_1 are the first stage constraint matrices for the variables in the vectors x_1 and y_1, respectively; x_t^ω and y_t^ω are the vectors of the second stage 0-1 and continuous variables, respectively, at time period t for scenario ω, for $t\in\mathscr{T}^-$, where $\mathscr{T}^- \equiv \mathscr{T}\backslash\{1\}$ and $\omega\in\Omega$; a_t^ω and b_t^ω are the vectors of the objective function coefficients for the variables in the second stage vectors x_t^ω and y_t^ω, respectively; $A_1^{t\omega}$ and $B_1^{t\omega}$ are the second stage constraint matrices for the variables in the vectors x_1 and y_1 ; $A_{t'}^{t\omega}$ and $B_{t'}^{t\omega}$ are the second stage constraint matrices for the variables in the vectors $x_{t'}^\omega$ and $y_{t'}^\omega$, respectively, for $1 < t' \le t$, $t\in\mathscr{T}^-$, $\omega\in\Omega$; h_t^ω is the rhs; $nx(1)$ and $ny(1)$ are the number of the first stage 0-1 and continuous variables, respectively; and $nx(t)$ and $ny(t)$ are the number of the second stage 0-1 and continuous variables, respectively, for $t\in\mathscr{T}^-$.

The model RN (1) aims to maximize the objective function expected value (i.e., mean) alone. The main criticism that can be made to this very popular strategy, as stated above, is that it ignores the variability of the objective function value over the scenarios for the RN solution and, in particular in our case, the "left" tail of the non-wanted scenarios. Strategy TSD requires a set of profiles $\{p\}$, say $\mathscr{P}_t\, \forall t\in\tilde{\mathscr{T}}$, given by the tupla $(\phi^p, S^p, \bar{s}^p\, \bar{\nu}^p)$, where ϕ^p is the objective function threshold to be satisfied up to time period t, S^p is the maximum *target* for the shortfall that is allowed on reaching the threshold up to time period t, $\bar{s}^p$ is the upper bound *target* of the expected shortfall on reaching the threshold, and $\bar{\nu}^p$ is the shortfall probability bound *target*. The model can be expressed as follows,

$$
z_{TSD} = \max\{a_1x_1 + b_1y_1 + \sum_{\omega\in\Omega} w^\omega \sum_{t\in\mathscr{T}^-} (a_t^\omega x_t^\omega + b_t^\omega y_t^\omega) - \sum_{t\in\tilde{\mathscr{T}}}\sum_{p\in\mathscr{P}_t} (M_S^p\varepsilon_S^p + M_{\bar{s}}^p\varepsilon_{\bar{s}}^p + M_{\bar{\nu}}^p\varepsilon_{\bar{\nu}}^p)\}
$$

$$
\begin{array}{lll}
\text{s.t.} & A_1x_1 + B_1y_1 = h_1 & \\
& A_1^{t\omega}x_1 + B_1^{t\omega}y_1 + \sum_{t'\in\mathscr{T}^-:t'\le t} (A_{t'}^{t\omega}x_{t'}^\omega + B_{t'}^{t\omega}y_{t'}^\omega) = h_t^\omega & \forall t\in\mathscr{T}^-,\ \omega\in\Omega \\
& x_1 \in \{0,1\}^{nx(1)},\ y_1 \in \mathbb{R}^{ny(1)} & \\
& x_t^\omega \in \{0,1\}^{nx(t)},\ y_t^\omega \in \mathbb{R}^{ny(t)} & \forall t\in\mathscr{T}^-,\ \omega\in\Omega \\
& a_1x_1 + b_1y_1 + \sum_{t'\in\mathscr{T}^-:t'\le t} (a_{t'}^\omega x_{t'}^\omega + b_{t'}^\omega y_{t'}^\omega) + s^{\omega p} \ge \phi^p & \forall p\in\mathscr{P}_t,\ t\in\tilde{\mathscr{T}},\ \omega\in\Omega \\
& 0 \le s^{\omega p} \le S^p\nu^{\omega p} + \varepsilon_S^p,\ \nu^{\omega p}\in\{0,1\},\ \varepsilon_S^p \in \mathbb{R}^+ & \forall p\in\mathscr{P}_t,\ t\in\tilde{\mathscr{T}},\ \omega\in\Omega \\
& \sum_{\omega\in\Omega} w^\omega s^{\omega p} \le \bar{s}^p + \varepsilon_{\bar{s}}^p,\ \varepsilon_{\bar{s}}^p \in \mathbb{R}^+ & \forall p\in\mathscr{P}_t,\ t\in\tilde{\mathscr{T}} \\
& \sum_{\omega\in\Omega} w^\omega \nu^{\omega p} \le \bar{\nu}^p + \varepsilon_{\bar{\nu}}^p,\ \varepsilon_{\bar{\nu}}^p \in \mathbb{R}^+ & \forall p\in\mathscr{P}_t,\ t\in\tilde{\mathscr{T}},
\end{array}
\tag{2}
$$

where $s^{\omega p}$ is the shortfall (continuous) variable that, obviously, is equal to the difference (if it is positive) between threshold ϕ^p and the objective function value up to time period t for $p\in\mathscr{P}_t$, $t\in\tilde{\mathscr{T}}$ for scenario $\omega\in\Omega$, $\nu^{\omega p}$ is a 0-1 variable such that its value is 1 if the objective function value up to time period t is smaller than threshold ϕ^p and otherwise, 0, and ε_S^p, $\varepsilon_{\bar{s}}^p$ and $\varepsilon_{\bar{v}}^p$ are the non-negative slack variables that take the violations of the S^p-, $\bar{s}^p$- and $\bar{v}^p$-bounds, respectively, being M_S^p, $M_{\bar{s}}^p$ and $M_{\bar{v}}^p$ the related big enough M-parameters for the penalization of that slack variables in the objective function. Observe that the time stochastic dominance strategy-based model object consists of controlling the objective function value at modeler-driven periods, instead of only performing it at the end of the time horizon. It is very useful for applications with long horizons.

Notice that the mixture of the FSD and SSD related constraints avoids the drawback of not having control on the expected shortfall over the scenarios up to given time periods in case of considering solely FSD. It also avoids the drawback of not having control on the fraction of scenarios with shortfall on reaching the thresholds in case of considering solely SSD.

3. TSD-FRCA2: a Fix and Relax Coordination algorithm for two-stage stochastic mixed 0-1 problems under TSD strategy

The main characteristic of the inexact approach that is proposed consists of considering so-named levels along the time horizon included by disjoint sets of consecutive periods. At each level an independent partial scenario mixed 0-1 model is solved by fixing the 0-1 variables of ancestor levels to the value obtained at the optimization of their related models and relaxing the integrality of the 0-1 variables related to successor levels as well as the (cross scenario SDC systems) for those other levels. Additionally, the continuous variables are obtained at each level by considering the bound targets of the SDC systems up to that level.More formally, the algorithm is formally described by:

Step 0: (**Initialization**). Parameter *mback*.

Step 1: (**Upper bound $\bar{z}$ of solution value z_{TSD}**).
Solve independently all the $|\Omega|$ scenario submodels (**??**).
If any of those submodels is infeasible then the original problem (2) is infeasible as well, STOP.
Set $z_{WS} := \sum_{\omega\in\Omega} w^\omega z_{ws}^\omega$ as expression (**??**) and upper bound $\bar{z} := z_{WS}$.
Solve the two-stage submodel (**??**).
If it is infeasible then the original problem (2) is infeasible as well, STOP.
Update $\bar{z} := \min\{z^1, z_{WS}\}$.
Set *nback* $:= 0$ and level $\ell = 2$.

Step 2: (**Forward step**)
If $\nexists t \in \tilde{\mathscr{T}} : t_f^\ell \leq t \leq t_e^\ell$ then solve independently all the $|\Omega|$ scenario RN models (**??**);
else solve model (3) TSD1

Step 3: (**Detecting infeasibility**)
If any RN model or the modeler-driven TSD1 or TSD2 model is infeasible, then:
If $nback = mback$ or $\ell = 2$ then STOP: "Original problem status: unknown"; else go to Step 4.
If $\ell = 2$ and option TSD1 is used then reset $\bar{z} := \min\{z_{TSD1}^\ell, \bar{z}\}$.
If $\ell = |\mathscr{L}|$ then go to Step 5.
Reset $\ell := \ell + 1$.
Go to Step 2.

Step 4: (**Backward step**)
Redefine the partition structure: $t_e^{\ell-1} := t_e^\ell, t_f^i := t_f^{i+1}, t_e^i := t_e^{i+1} \forall i = \ell, ..., |\mathscr{L}| - 1, |\mathscr{L}| :=$

$|\mathscr{L}|-1$.
Reset $nback := naback + 1$ and $\ell := \ell - 1$.
Go to Step 2.

Step 3: (**Save TSD solution**).
Save $x_1 = \hat{x}_1,\, y_1 = \hat{y}_1,\, x_t^\omega = \hat{x}_t^\omega,\, y_t^\omega = \hat{y}_t^\omega \,\forall t \in \mathscr{T}^-,\, \omega \in \Omega$.
Save upper bound $\bar{z}$.
Compute solution value $\underline{z} = a_1\hat{x}_1 + b_1\hat{y}_1 + \sum_{\omega\in\Omega} w^\omega \sum_{t\in\mathscr{T}^-}(a_t^\omega \hat{x}_t^\omega + b_t^\omega \hat{y}_t^\omega)$.
Compute TSD bounds for all $p \in \mathscr{P}_t, t \in \tilde{\mathscr{T}}$; $\hat{S}^p = \max_{\omega\in\Omega} \hat{s}^{\omega p}$, $\hat{\bar{s}}^p = \sum_{\omega\in\Omega} w^\omega \hat{s}^{\omega p}$ and $\hat{\bar{\nu}}^p = \sum_{\omega\in\Omega} w^\omega \hat{\nu}^{\omega p}$ from the solution, and STOP.

where the model (3) is given by:

$$z_{TSD1}^\ell = \quad a_1\hat{x}_1 + b_1\hat{y}_1 + \max\{\sum_{\omega\in\Omega} w^\omega \sum_{t\in\mathscr{T}^-}(a_t^\omega x_t^\omega + b_t^\omega y_t^\omega) - \sum_{t\in\tilde{\mathscr{T}}:t_f^\ell\le t\le t_e^\ell}\sum_{p\in\mathscr{P}_t}(M_S^p\varepsilon_S^p + M_{\bar{s}}^p\varepsilon_{\bar{s}}^p + M_{\bar{\nu}}^p\varepsilon_{\bar{\nu}}^p)\}$$

$$\begin{array}{lll}
\text{s.t.} & A_1^{t\omega}\hat{x}_1 + B_1^{t\omega}\hat{y}_1 + \sum_{t'\in\mathscr{T}^-:t'\le t}(A_{t'}^{t\omega}x_{t'}^\omega + B_{t'}^{t\omega}y_{t'}^\omega) = h_t^\omega & \forall t\in\mathscr{T}: t_f^\ell \le t,\, \omega\in\Omega \\
& x_t^\omega = \hat{x}_t^\omega,\, y_t^\omega = \hat{y}_t^\omega & \forall t\in\mathscr{T}^-: t < t_f^\ell,\, \omega\in\Omega \\
& x_t^\omega \in \{0,1\}^{nx(t)},\, y_t^\omega \in I\!R^{ny(t)} & \forall t\in\mathscr{T}: t_f^\ell \le t \le t_e^\ell,\, \omega\in\Omega \\
& x_t^\omega \in [0,1]^{nx(t)},\, y_t^\omega \in I\!R^{ny(t)} & \forall t\in\mathscr{T}: t > t_e^\ell,\, \omega\in\Omega \\
& a_1\hat{x}_1 + b_1\hat{y}_1 + \sum_{t'\in\mathscr{T}^-:t'\le t}(a_{t'}^\omega \hat{x}_{t'}^\omega + b_{t'}^\omega y_{t'}^\omega) + s^{\omega p} \ge \phi^p & \forall p\in\mathscr{P}_t,\, t\in\tilde{\mathscr{T}}: t_f^\ell\le t\le t_e^\ell,\, \omega\in\Omega \\
& 0 \le s^{\omega p} \le S^p\nu^{\omega p} + \varepsilon_S^p,\; \nu^{\omega p}\in\{0,1\} & \forall p\in\mathscr{P}_t,\, t\in\tilde{\mathscr{T}}: t_f^\ell\le t\le t_e^\ell,\, \omega\in\Omega \\
& \sum_{\omega\in\Omega} w^\omega s^{\omega p} \le \bar{s}^p + \varepsilon_{\bar{s}}^p,\, \varepsilon_{\bar{s}}^p \in I\!R^+ & \forall p\in\mathscr{P}_t,\, t\in\tilde{\mathscr{T}}: t_f^\ell\le t\le t_e^\ell \\
& \sum_{\omega\in\Omega} w^\omega \nu^{\omega p} \le \bar{\nu}^p + \varepsilon_{\bar{\nu}}^p,\, \varepsilon_{\bar{\nu}}^p \in I\!R^+ & \forall p\in\mathscr{P}_t,\, t\in\tilde{\mathscr{T}}: t_f^\ell\le t\le t_e^\ell.
\end{array} \quad (3)$$

4. Computational tests

5. Conclusions

6. Acknowledgement

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