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METHODOLOGIES AND APPLICATION

Designing closed-loop supply chains with nonlinear dimensioning factors using ant colony optimization

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Abstract Closed-loop supply chain (CLSC) design implies the modelling of the forward and the reverse flows of products in an integrated way. This paper introduces nonlinear dimensioning factors in the design of CLSC and uses ant colony optimization to optimize the design of the supply chain. The proposed algorithm is called SCAnt-NLDesign. The modelled nonlinear dimensioning factors are: cost variations in transportation distances between facilities (tapering principle), scale economies related to transported quantities, and scale economies regarding the facilities' capacity. Results show that the proposed SCAnt-NLDesign algorithm reduced the total cost in 44 %, when compared to a linear formulation of a CLSC. Note also that a mixed integer linear programming implementation of the nonlinear CLSC was not able to get closed to the optimal solution, given worse results than the linear CLSC.

Keywords Supply chain · Closed-loop supply chain · Ant colony optimization · Nonlinear dimensioning factors

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1 Introduction

As any business activity, the objective of a supply chain is to maximize the generated economic value (Chopra and Meindl 2014) by satisfying its customers' requirements with products and services from multiple linked suppliers. A supply chain (SC), at a strategic/tactical level, is defined as a life cycle process comprising physical, information, financial, and knowledge flows (Ayers 2010), between its composing entities. Activities such as procurement, production, warehousing, storage, transportation and demand management are the main flow generators through the planned structure. The adequate design of a SC requires the anticipation of these flow levels (Klibi et al. 2010). The effective coordination of these flows, within the scope of the defined supply chain objective(s), is the main goal of supply chain management (SCM) (Harrison et al. 2004). SCM then is a process that aims for efficiency , ranging from the design, planning, implementation and control of all the activities (Melo et al. 2009). SCM spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption.

More recently, due to the increase in environmental consciousness and legislative obligations of companies, SCM no longer stops at the point-of-consumption, but has been expanded to consider this stage as a new generator of products, these are among others, the finished goods, referred previously, after their use by the point-of-consumption entities (usually the customers) (Fleischmann et al. 2005). This flow of products has been showed to be a good source of revenues to companies since, after recycling they can be re-introduced in the market. When these two supply chains (forward—the supply and reverse—the return) are considered as an integrated structure, this is called a closed-loop supply chain, and consequently its design must be approached in an integrated way (Salema et al. 2010), and more recently in Esteves et al. (2012), where a mixed integer linear programming (MILP) model and an ant colony optimization (ACO) algorithm application, respectively, are used to design a closed-loop supply chain (CLSC).

The supply chain design, in the context of this work is done by selecting the feasible set of factories, warehouses and disassembly centres, and the flows of products between each one of these facilities that optimizes a predetermined criteria for value creation.

Baumgartner et al. (2012) showed that the inclusion of nonlinear conditions at the strategic stage leads to better supply chain designs. The introduction of these nonlinear factors is important due to the generation of economical scale effects shown to have a significant impact on the global cost of the modelled supply chain and on the network structure. Therefore, in this work, nonlinear criteria factors are considered for the supply chain design.

The nonlinearities considered in this paper are the most common in SCM when modelling the scale economy effects (Train and Wesley 2007; Baumgartner et al. 2012): scale economies with transportation distance—the tapering principle; scale economies with transported quantities—cost/transported quantity coefficient and warehouses and disassembly centres cost-capacity factor.

These factors provide not only a new and more realistic approach to CLSC design, but, at the same time, test the capacity of MILP formulations to solve this kind of problems, in which the introduction of nonlinearities renders the formulation much more complex.

This paper proposes an ant colony algorithm to optimize closed-loop supply chains entitled SCAnt-NLDesign. This paper proposes an ant colony algorithm to optimize CLSCs entitled SCAnt-NLDesign. Three types of nonlinearities are considered: scale economies with transportation distance (tapering principle); scale economies with transported quantities (cost/transported quantity coefficient); and warehouses and disassembly centres' cost-capacity factor. The simultaneous formulation of these three nonlinearities is novel in the supply chain formulation, to the best of our knowledge. Although the proposed algorithm is based on the SCAnt-Design algorithm proposed in Esteves et al. (2012), SCant-NLDesign introduces new pheromone matrices, and a new approach to optimize them, as well as different heuristics which are defined in a clear mathematical way (see Sect. 5.2.2). This paper shows clearly the advantage of SCAnt-NLDesign over the MILP formulation in terms of results and computational time.

Section 2 gives an overview of the supply chain management process and reviews the literature related to this work. Section 3 presents a definition and more detailed characterization of the supply chain problem, narrowing the scope and defining the modelling principles that conditioned Sect. 4, in which the mathematical model is structured and presented. This model was the base for Sect. 5, in which the developed ACO algorithm and its implementation is described. The application of the developed algorithm to a case study is presented in Sect. 6. The dataset used and the results achieved are presented as well as a benchmarking of the results, by comparing them to other methods. In Sect. 7, the conclusions achieved with this work, and possible future work is presented.

2 Supply chain management and literature review

SCM can be defined as the systemic, strategic coordination of the traditional business functions within a particular company and across businesses within the supply chain, for the purposes of improving the long-term performance of the individual companies and the supply chain as a whole (Hugos 2006). It complies the coordination of production, inventory, location, and transportation among the participants in a supply chain to achieve the best mix of responsiveness and efficiency for the market being served (Mentzer et al. 2001).

As any management activity, the decisions regarding SCM can be classified in levels, according to their time horizon, financial investments among others. The decisions classified as strategic are made for a long time horizon (many months or years), with high financial investments, based on predictions and assuming very little or no uncertainty in data (Harrison et al. 2004). In the context of SCM, the strategic component is usually named supply chain design and includes the following decisions (Harrison et al. 2004; Melo et al. 2009), which are made with the goal of satisfying customers' demands:

- Which facilities should be used (opened)?
- What production processes?
- What transportation modes and lanes?
- What size should the workforce be?
- Which customers should be serviced from which facility (facilities) so as to minimize the total costs?
- Definition of internal policies for relationships between the participants.

In Govil and Proth (2002) it is noticed that when answering to these questions, one should account for two cases, the case in which all the components belong to the same company and the case in which some of the components do not. In the first case, the goal is to maximize the total profit generated by the system, even if the decisions to reach such goal result in an increase in the cost of some activities. In this paper, the case study presented in Sect. 3 considers the existence of only one owner.

The decisions made at strategic level, consider an "intensive" use, during the time horizon, and will act as constraints for the tactical/operational level decisions. This intensive use is called the supply chain execution, which according to Harrison et al. (2004), deals with tactical and operational issues and is focused on the definition and implementation of short-term (from days to months) plans for the accomplishment of its objectives. These plans are made based on data that are assumed to vary according a certain probability distribution and with the constraints of the pre-established and fixed (or nearly so) infrastructure. Since we are working at a strategic level, the cost associated with the modification of a decision is very high. The concept of dynamics is recognized but not directly considered, instead the model will consider the demand as stable for the time horizon and assume robustness in the supply chain design.

2.1 Literature review

2.1.1 Supply chain design optimization

One of the most usual approaches to make a supply chain design is through the use of operations' research methodologies, that have the intent of "optimizing" a representation of the supply chain, normally through a graph approach, in the scope of one or several objectives and constraints. The literature concerning the supply chain design is extensive. Melo et al. (2009) made a comprehensive literature survey of works related to location models in the context of SCM. In this work, the main features to be considered in support decision making in the SCD context were identified. Authors argue that most models considering several supply chain design features do it in a simplified way, lacking on real context. Barbosa-Póvoa (2012) reviews works published since 2008 addressing supply chain optimization. Among future challenges, author points out the need for developing solution methods to solve large-scale problems.

The integration of nonlinear issues into supply chain modelling has been scarcely done. To overcome one of the major nonlinear models drawbacks, finding global optima, linearization approaches have been followed. For instance, nonlinear capacities' costs have been formulated by modular capacities (e.g., Correia and Captivo 2003), nonlinear production costs as piecewise linear function (e.g., Boek et al. 2006), among others. Very recently, Baumgartner et al. (2012), presented the design of a forward supply chain where they introduced economies of scale in transportation and storage costs, namely product cost and transportation cost. They used interactive linearization techniques to deal with the nonlinear costs which were used in the heuristics developed—deterministic relaxed mixed integer programming (MIP) drop heuristic and deterministic dynamic slope-scaling drop. The results obtained were compared to the ones of a branch-and-bound. They showed the superiority of heuristic approaches to design a multi-echelon forward supply chain, even though the MIP-based heuristic became impossible to use when in the case of very large problems.

2.1.2 Closed-loop supply chain design optimization

Aras et al. (2010) review models for the supply chain design where reverse flow of products are contemplated (including CLSC models). Authors argue that CLSC design has deserved less attention by academia than other reverse supply chain models.

Salema et al. (2010), proposed a methodology for the simultaneous design and planning, in which the two decision levels are integrated by the use of time. A period in the strategic level of design is considered as a sum of periods (smaller) at a tactical level. With the objective of minimizing the global costs and considering environmental limitations, a MILP formulation is presented. This formulation allows the modelling of more realistic features of supply chains, including flows' travel times, facilities processing times, product bill-of-materials and product disassembly structures. Pishvaee et al. (2011) propose the optimization of a CLSC using a MILP model for the design of the supply chain, and posteriorly its robustness is assessed by comparing it to other solutions obtained by making variations in the models parameters. This paper considers and also presents the existence of two markets: one for new products' demand and other for remanufactured products' returns. Khajavi et al. (2011), proposed a bi-objective MILP model, for the design of an integrated, forward and reverse, multi-echelon supply chain. The defined objectives were the minimization of costs and the maximization of the network responsiveness using a branch-and bound technique. The authors refer that their work differs from the other bi-objective models because it addresses a forward and reverse integrated supply chain. Cardoso et al. (2013) developed a MILP model for the CLSC design where flows between all facilities are modelled, including transshipment flows. Demand uncertainty is also modelled together with facility capacity expansion.

The main disadvantages of MILP, when compared to other approaches are the lack of detail that can be included in a model that still allows a "fluid" solving of the problem and, the bigger amount of data needed to characterize the system (Esteves et al. 2012).

2.1.3 Metaheuristics for supply chains design

Combinatorial optimization problems consist of analysing a mathematical problem by searching a discrete variable solution space, to find a solution that best satisfies the objectives of the problem being solved. The number of possible solutions associated with these problems is usually so big that an exhaustive search is very expensive, turning this approach unfeasible. To surpass this limitation, the use of metaheuristics is made. This allows the lowering of the computational cost, at the expense of not having the guarantee of achieving an optimal solution due to the "trap" of local optima (Dorigo and Stützle 2004).

A metaheuristic is a structured procedure that acts as an orchestrator of iterations between local improvement procedures and higher level strategies to enable the capacity of escaping local optima and enhancing the search of a solution space (Glover and Kochenberger 2003).

Chen et al. (2012) made a survey of several metaheuristics, more specifically the one's that emulate or are inspired by living beings and used in the modelling and optimization of SCM systems. Among these, the authors identified genetic algorithms, evolutionary programming, evolution strategies, differential evolution, artificial immune and swarm intelligence, which can be sub-divided in particle swarm, artificial bee colonies and ant colonies.

ACO (Dorigo and Stützle 2004) is an algorithm that mimics the behaviour of some species of ants. The natural coordination mechanism among the several ants of the same colony works in an indirect way and is called stigmergy. The reaction of the same or other ants to the pheromones left in the environment generates a medium of indirect communication that, in time, promotes the emergence of a trend to adopt the "optimal" length path (among the ones available) between the colony and food sources.

In ACO, artificial ants roam freely throughout the search space while building stochastic solutions derived from probabilistic decisions based on possibly available heuristic information on the problem in question and artificial pheromones, which change dynamically at run-time to reflect the agents' acquired search experience. In case heuristic information is used, one can interpret ACO as being an extension of traditional construction heuristics through the integration of the dynamically changing artificial pheromones (Dorigo and Stützle 2004; Glover and Kochenberger 2003).

The application of ACO techniques to SCM has been mainly at tactical and operational levels. Chen et al. (2012), made a review of these applications, as e.g., Silva et al. (2009) considers a multi-actor supply chain in which an ACO algorithm allows the exchange of information between different optimization problems by means of a pheromone matrix, to allow for the multiple actors to optimize their own performance while at the same time being part of a bigger system. Wang (2009) dealt with supply chains with losses of production called defective supply chains. In Wang (2009), there is no mention to the question of reverse logistics.

In terms of application of ACO to design, Chen et al. (2012) refers Moncayo-Martínez and Zhang (2011), in which a multi-objective ACO algorithm is presented. The algorithm has the simultaneous objective of minimizing the costs and the accomplishment of the delivery due-dates. The analysed problem involved the choice of the resources across the supply chain network. A multi-objective ACO, is proposed in which a multi-colony is used. The successive determination of non dominated solutions for each of the colonies will form the final non-dominated solution set.

Esteves et al. (2012), proposed an ACO algorithm that had the objective of designing a CLSC. This work included multi-product and multi-echelon considerations. The algorithm not only served as a decision support tool for the facilities location, but also the simultaneous assignment of flows to a particular facility. In terms of supply chain design it is the only ACO application found, that contemplates closed-loop supply chains. Beyond any ACO for CLSC design this paper introduces nonlinear factors, and to the best of our knowledge no other, making the same considerations was published to the date.

3 Problem description and characterization

Nowadays, a supply chain has as a central actor for its design, the customer as the generator, not only of demand (forward chain), but also acting as a supplier (reverse chain) of returned product. This satisfaction implies the existence of flows of products, variable in quantity and type, between several entities each with its own function(s) (processes—production, inventory, logistics and distribution, reprocessing, disposal or even generation of demand and return) and consequent influence in the state (characteristics) of the product flow between them.

The supply chain entities considered in this paper are factories, warehouses, disassembly centres, disposal facilities and customers. For instance, a flow from a factory to a warehouse, has its own characteristics, namely type of products, quantities and unitary transportation costs.

To be appealing from a business perspective, the satisfaction of customer needs must be economically advantageous, i.e. the costs must be at least equal or inferior to the gains or profits achieved with the supply chain. To evaluate the economical viability of the investment it is necessary, among others, to analyse the costs incurred and what is the minimal total value of these (in this case, only at a strategic level) that allow the company to satisfy its objectives.

The objective of minimizing the global supply chain cost will imply the determination of the network's characteristics, more specifically, the facilities that compose it and the flows between the different types of facilities and to the customers. A possible conceptual model for the considered supply chain is represented in Fig. 1, where N_p is the number of factories, N_w the number of warehouses, N_r the number of disassembly centres, and N_c the number of customers.

Table 1 describes the data needed to design a supply chain. This information, is usually related to the supply chain per

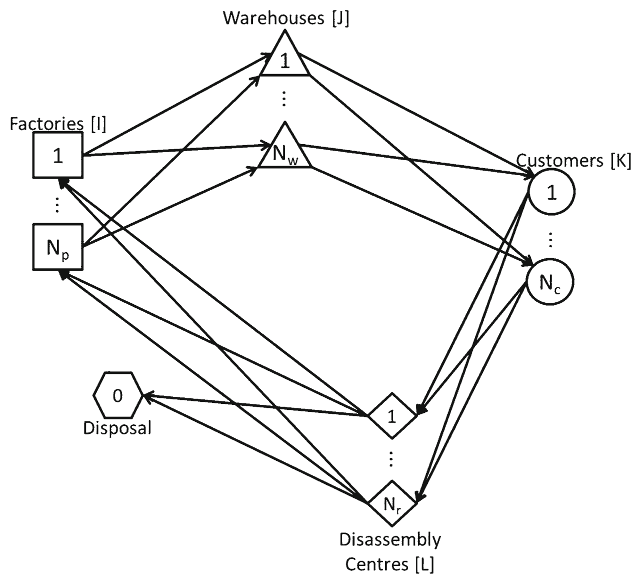

Fig. 1 Supply chain conceptual model

Table 1 Supply chain data

Minimal disposal fraction	
Customer's demand volume	Customer data
Customer's return volume	
Customer's geographical data	
Customer's non-satisfaction costs	
Type or kind: relates to the entity's function	Facility data
Geographical data	
Opening or renting costs	
Capacity: maximum/minimum quantity of products that can be processed	
Type of products	Flow data
Flow capacity	
Unitary transportation costs	

se and the possible external conditioners, as e.g. political or legislative restrictions.

This model considers that there are some customers' demand or return not economically viable to satisfy. Therefore, to be able to analyse and compare both the alternatives, a non-satisfaction cost is associated with each customer. This non-satisfaction costs are directly related to a measure of the "service level" that the company wants to assure to its customers, and reflect their importance to the company. Higher costs will imply the need to maximize the number of satisfied customers (Salema et al. 2010).

3.1 Supply chain model

A supply chain can be modelled as an edge-weighted directed graph $G(V, A, X)$, where each of the nodes (V) represents an entity (factory, warehouse, disassembly centre, customer or other) and the weight (X) of each edge (A) a quantification of the referred flow.[1]

The SC model in this paper is based on the one presented in (Salema et al. 2007), for a CLSC. The supply chain is divided in echelons according to the nodes nature. The echelons in this supply chain are:

1. Factories → warehouses;
2. Warehouses → customers;
3. Customers → disassembly centres;
4. Disassembly centres → factories.

This is an important classification, since it is considered that the flows only occur in the same echelon, i.e., a flow cannot connect a warehouse to a disassembly centre. The model has the objective of optimizing both chains simultaneously.

This work, makes use of the SCant-Design algorithm (Esteves et al. 2012), expanding it through the accounting of possible economies of scale in the supply chain, giving it not only a more realistic approach, but also, demonstrating the capacity of metaheuristic methods in solving more complex supply chain problems than the traditional linear and integer approaches like MILP (Salema et al. 2007).

The SC model has the objective of defining the number, location and capacity of the facilities to be opened, and also, the product type and quantity to be processed by each of the facilities. This SC model can be used to solve location-allocation, nonlinear, multi-echelon and multi-product problems.

3.1.1 Supply chain representation

The supply chain presented in Fig. 1 can be modelled as a network. With the purpose of representing the closed-loop supply chain the most realistically possible, the following network elements are considered:

– The nodes (facilities) have the following characteristics:

Input flows—arcs that "bring" products from the previous network level;
Output flows—arcs that "send" products to the next network level;
Process—change in the flow characteristics, in this paper, only quantities can be subjected to changes;

[1] A directed graph, also known as a network, is a finite set of nodes and a set of edges that are defined as ordered pairs of nodes. This order implies a one-way connection between nodes. If a scalar weight is associated with every edge the directed graph is called edge-weighted (Christou 2012).

Capacity—production or storage amounts associated with each facility.

– The arcs (flows) are characterized by:

Origin—source entity;
Destination—target entity;
Product—each flow has one product associated with it;
Capacity— related to transportation, define the limits of each flow.

3.1.2 Modelling assumptions

In the modelling process, the following assumptions were considered:

Flow of products—there is no flow of materials between facilities of the same type (e.g. factory → factory).
Stability of the parameters in the time horizon—it has been considered that the parameters involved in the modelling are stable and do not change in the time horizon.
Value and strategic managerial weight of the customer—due to the available data for the case study, this is modelled by considering associated with each customer, a cost of non-satisfaction for its demand and return.
Storage capacity limits—it has been considered that the warehouses and disassembly centres have a minimum and maximum storage capacity. Bellow the minimum the facility is not a viable investment. The maximum is the top of the budget allowed for the investment in each facility.
Inventory costs—are considered null, since the warehouses and disassembly centres are considered as transshipment platforms, with a zero inventory policy.
Realistic storage capacities—for warehouses and disassembly centres, the total planned capacity is equal to the quantities that will pass through the facility rounded to the next minimum capacity multiple defined for the problem.
Single transportation mode—a single transportation mode has been assumed for the model. This transportation mode has limited capacity implying a maximum shipment size imposed on all network flows.

3.2 Nonlinearities in supply chain design

The nonlinearities considered in this paper are the ones that were identified as the most common in SCM (Train and Wesley 2007; Baumgartner et al. 2012). As referred in Baumgartner et al. (2012) the inclusion of nonlinear conditions at the strategic stage leads to better supply chain designs and non-considering them is translated into a lack of realism of the model. The introduction of these nonlinear factors is important due to the generation of the economical scale effects shown in Sect. 4 and have a significant impact on the global cost of the modelled supply chain.

These nonlinearities are the following:

– Scale economies with transportation distance—the tapering principle;
– Scale economies with transported quantities—cost/transported quantity coefficient;
– Warehouses and disassembly centres' cost-capacity factor.

In Train and Wesley (2007), it is referred that to consider, in a realistic way, the effects of distance in the location choices of facilities, the relationship between transportation costs and distance must be defined. This relationship is usually expressed in terms of cost per transported unit, which is also a function of the distance. The transportation price per quantity shipped is monotone increasing and concave with the distance, this nonlinear variation is usually referred in the literature as the tapering principle.

The tapering rates are defined as a non-direct proportional increase in rates with distance, this is due to the ability of a company to spread the transportation costs over a great number of miles (Coyle et al. 2008). This principle is referred in Rodrigue et al. (2006), Sahin et al. (2009) and Forkenbrock (1999). Forkenbrock (1999) analysed several cost generators in trucking companies, and showed that the private operating costs condition the unit cost per distance of transporting cargo. This unit cost shows a decreasing tendency with the total length of the route chosen for transportation.

In Lapierre et al. (2004), it is showed the existence of economies of scale at the operational/tactical level, by the improvement of the transported loads to their maximum values. This cost variation is showed in Baumgartner et al. (2012) and defined as a nonlinear function of the transported volume, due to crescent discounts—incremental discount—with the increasing of transported volume.

Assuming a fixed cost for a facility, independently of its capacity, or a linear variation of the cost with capacity is not very realistic. When the projects are similar, it has been shown that the ratio of their capacities is not usually the same as the ratio of their costs (Remer and Mattos 2002). The cost of opening a warehouse or a disassembly centre, is then a function of the capacity defined for the facility. This capacity is the total demand or return to be served by that facility.

This methodology is normally used to estimate the cost of a facility, in a very early project stage, when the information is very scarce and the required precision is very raw. An estimation (order of magnitude) of the costs to be incurred can be produced by using historical information of other similar projects. An adjustment of these project's characteristics (time, location and capacity) to those of the future facility, through the use of correctional indexes. Remer and Mattos (2002), identified the following indexes: inflation indexes; location indexes and capacity indexes, indicating that due to their nature, the inflation and location indexes generate a linear variation in the costs while the capacity index is nonlinear. tIn this work, only the capacity index is considered, due to the difficulty in estimating the other ones.

The modelling of these factors assumed, as in Salema et al. (2007), that each flow has one product associated with it. Meaning that if there are two materials flowing between the same pair of origin and destination, a different flow is defined. Assuming this, the following economies of scale and nonlinearities are considered.

3.2.1 Scale economies with transportation distance: the tapering principle

Forkenbrock (1999), made an analysis of the several expense categories incurred by a private trucking company, like salaries, wages, taxes and licenses, insurance and others, and the length of haul. The result of this analysis led to the establishment of three distance intervals with characteristic unitary transportation costs. These data were used in this paper for the establishment of a cost variation model that could be representative of this principle. The analysis of the available data showed that the influence of this effect can be included as a correctional factor Ψ_t^c to be applied to the distance, conditioning in this way the transportation cost. This correctional factor Ψ_t^c is dimensionless and can be expressed as:

$$\Psi_t^c = \alpha_t \cdot e^{\beta_t \cdot \left(\frac{t}{t_{\max}}\right)} + z, \tag{1}$$

where ($t_{\max}$) is the maximum distance between any two entities of a supply chain, t is the distance between the facilities being analysed, α_t and β_t are parameters that fit the real data and z the value to which non-considerable variations of the cost ratio occur. A detailed explanation of the methodology adopted for the determination of these parameters is presented in Sect. 4.2.1.

3.2.2 Scale economies with transported quantities: cost/transported quantity coefficient

In this work, the transportation frequencies are not considered directly, but emulated considering a maximum limit to each shipment size. The model formulation will condition these transportation frequencies by trying to find the best solution in which one of the components of the correctional factor for the transportation costs induces a fixed amount. The lowest cost supply chain network topology implies, among others, the minimization of the number of shipments.

Considering the analysis presented in Lapierre et al. (2004), for the less than truck load shipment costs, in a certain geographical area and the following variables:

TW_m	Total weight of product m to be transported;
W_m	Unitary weight of the product m;
C^{TW}	Unitary cost to transport TW_m;
$C^{TW}_{\max}$	Maximum unitary cost to transport TW_m;
X_m	Demand (quantity) of transported product m;
$Q_{\max}$	Maximum truck capacity, or delivery size;

and,

$$X_m = \frac{TW_m}{W_m}. \tag{2}$$

Let the correctional factor applied to unitary cost for any product m that accounts for the scale economies related to the transported quantities be Ψ_X^c, and can be defined as:

$$\Psi_X^c = \alpha_X + \beta_X \cdot \ln\left(\frac{X_m}{Q_{\max}}\right) \tag{3}$$

where α_X and β_X are parameters that fit the real data.

3.2.3 Warehouses and disassembly centres' cost-capacity factor

For the same type of facility there is a nonlinear relation between their costs and capacity (Remer and Mattos 2002):

$$\frac{f_f}{C_1} = \left(\frac{Q_2}{Q_1}\right)^{\ell} \tag{4}$$

where,

C_1	Cost of a base facility;
f_f	Cost of new facility f;
Q_1	Capacity of a base (existing) facility;
Q_2	Capacity of the future facility;
ℓ	Cost-to-capacity factor.

This nonlinear cost variation with capacity is of an exponential nature and the index Δ_q can be defined as (Zugarramurdi et al. 2002):

$$\Delta_q = \left(\frac{Q_2}{Q_1}\right)^{\ell} \tag{5}$$

The cost-to-capacity factor ℓ is conditioned by the type of industry considered (Wibowo and Wuryanti 2007). In the case of supply chains, the value for warehouses and disassembly centres is $\ell = 0, 8$ (Peña-Mora et al. 2003). A base facility is a facility with exactly the same characteristics as the one to be established and for which, its capacity Q_1 and respective cost C_1 are known.

Using the previous Eqs. (4) and (5), the expression of the costs of new structures involved in this problem can be expressed as:

$$f_f = C_1 \times \Delta_q = C_1 \times \left(\frac{Q_2}{Q_1}\right)^{\ell}. \quad (6)$$

4 Problem formulation

4.1 Sets, parameters and variables

(a) Sets

The supply chain presented in this paper considers the following sets: $I = \{1, \ldots, N_p\}$ for potential factories, $I_0 = I \cup \{0\}$ for potential factories plus a disposal option, $J = \{1, \ldots, N_w\}$ for potential warehouses, $L = \{1, \ldots, N_r\}$ for potential disassembly centres, $K = \{1, \ldots, N_c\}$ for potential customers and $M = \{1, \ldots, N_v\}$ for product types.

(b) Parameters

To model the supply chain presented in Fig. 1, the following parameters have been considered:

- d_{mk} and r_{mk}—demand and return of product m for customer $k, k \in K, m \in M$;
- γ—minimal disposal fraction;
- $Q_{\max}$—maximum delivery size;
- Distances (km)

t_{ij}	Distance between factory i and warehouse $j,\ i \in I,\ j \in J$;
t_{jk}	Distance between warehouse j and customer $k,\ j \in J,\ k \in K$;
t_{kl}	Distance between customer k and disassembly centre $l,\ k \in K,\ l \in L$;
t_{li}	Distance between disassembly centre l and factory $i,\ l \in L,\ i \in I_0$;
$t_{\min}$	Minimal distance between any two facilities.

- *Unitary costs per km per transported unit:*

c^{f1}_{mij}	Unitary cost for product m between factory i and warehouse j in the forward flow;
c^{f2}_{mjk}	Unitary cost for product m between warehouse j and customer k in the forward flow;
c^{r1}_{mkl}	Unitary cost for product m between customer k and disassembly centre l in the reverse flow;
c^{r2}_{mli}	Unitary cost for product m between disassembly centre l and factory i in the reverse flow.

- *Facilities fixed opening costs:*

f^p_i	Fixed total cost of opening factory $i, i \in I$;
C^w_1	Cost of opening a base warehouse;
C^r_1	Cost of opening a base disassembly centre.

- *Warehouses and disassembly centres' opening costs:*

f^w_{jq}	Cost of opening warehouse $j, j \in J$;
f^r_{lq}	Cost of opening disassembly centre $l, l \in L$.

- c^u_{mk} Unit cost of customer k non-satisfied demand of product m;
- c^w_{mk} Unit cost of customer k non-satisfied return of product m;
- *Facility capacities:*

$g^p_i,\ t^p_i$	Maximum and minimum capacity of factory $i, i \in I$;
$g^w_j,\ t^w_j$	Maximum and minimum capacity of warehouse $j, j \in J$;
Q_{1j}	Capacity of a base warehouse;
$g^r_l,\ t^r_l$	Maximum and minimum capacity of disassembly centre $l, l \in L$;
Q_{1l}	Capacity of a base disassembly centre.

(c) Variables

X^{f1}_{mij}	Demand of product m served by factory i to warehouse j in the forward flow $f1$;
X^{f2}_{mjk}	Demand of product m served by warehouse j and customer k in the forward flow $f2$;
X^{r1}_{mkl}	Demand of product m returned by customer k to disassembly centre l in the reverse flow $r1$;
X^{r2}_{mli}	Demand of product m returned by disassembly centre l and factory i in the reverse flow $r2$;
U_{mk}	Non-satisfied demand amount of product m to customer k;
W_{mk}	Non-satisfied return amount of product m to customer k;
$Y^p_i = 1$	If factory i is opened, $i \in I$;
$Y^w_{jq} = 1$	If warehouse j is opened with a capacity q, $j \in J$;
$Y^r_{lq} = 1$	If disassembly centre l is opened with a capacity $q, l \in L$.

4.2 Nonlinearities modelling

To account for the nonlinearities defined in Sect. 2.1.2, the following correctional factors were considered in the model: If, Ψ^a_b is the correctional factor of the parameter (or variable)

a, considering the parameter (or variable) b, the following can be defined, respectively, for the tapering principle, economies of scale with transported quantities and warehouses and disassembly centres' building sizes.

4.2.1 Tapering principle

To define the correctional factor Ψ_t^c the following methodology was followed. The data presented in Forkenbrock (1999) were first turned dimensionless relatively to their maximum cost and deduced from the value after which independently of the distance increase, non-considerable variations of the cost ratio occur $-z$. Where z is the ratio between the minimum and maximum costs, adopted considering a conservative approach.

These values were associated to the minimum ($t_{\min}$), middle (t_{mid}) and maximum ($t_{\max}$) distances between any two identities of a supply chain, made dimensionless in relation to the maximum distance ($t_{\max}$). The resulting ordered pairs are the points presented in Fig. 2.

Using these data points, a regression was made to deduce the variation rule that was the best representation of the data. The following model for the unitary transportation cost per distance was found to be the one that best represents this variation, due to its high value of $R^2 = 0.9958$, which is an indicator of the goodness of fit of the model. The closest to 1, the better the regression lines approximates the real data points.

To this variation rule, the model was completed with the adding of the value of z.

This effect is included in the CLSC, as a correctional factor Ψ_t^c to be applied to the distance, conditioning this way the transportation cost. Applying the case study data, where the maximum distance between any two facilities is $t_{\max} = 200$ km and $z = 0.8$. This correctional factor Ψ_t^c can be expressed as:

$$\Psi_t^c = 0.214 \cdot \mathrm{e}^{-0.014 \cdot t} + 0.8 \qquad (7)$$

where $\alpha_t = 0.214$, $\beta_t = -2.8$.

The cost correctional factor Ψ_t^c variation is presented in Fig. 2. Considering (7) the variables presented in Table 2 were defined.

4.2.2 Economies of scale with transported quantities

After analysing the case study data, the maximum transported quantity between any two facilities is $Q_{\max} = 3{,}000$ units and $W_m = 1$ weight unit. Using the model presented in (3), and the case study data, the correctional factor Ψ_X^c is defined as:

$$\Psi_X^c = 0.2815 - 0.134 \cdot \ln\left(\frac{X_m}{3{,}000}\right) \qquad (8)$$

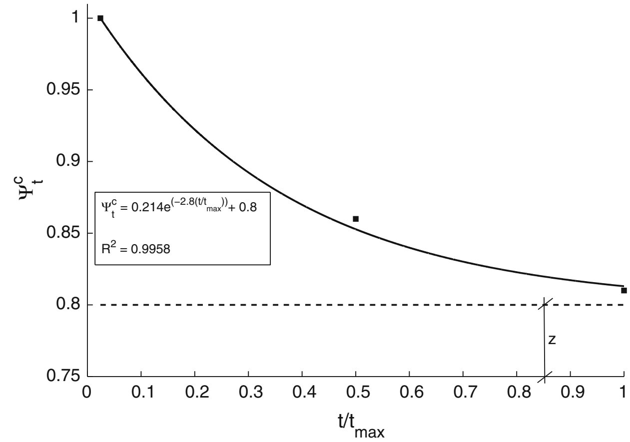

Fig. 2 Variation of the correctional factor Ψ_t^c with the distance

Table 2 Tapering principle parameters

	t	Ψ_t^c
Factory—Warehouse	t_{ij}	$\Psi_{t_{ij}}^c$
Warehouse—Customer	t_{jk}	$\Psi_{t_{jk}}^c$
Customer— Disassembly centre	t_{kl}	$\Psi_{t_{kl}}^c$
Disassembly centre—Factory	t_{li}	$\Psi_{t_{li}}^c$

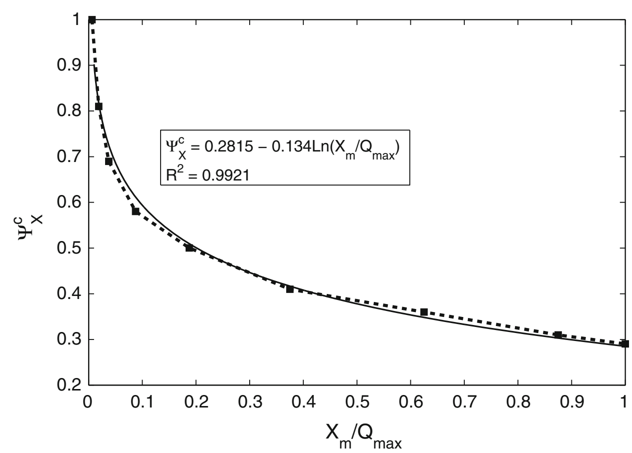

Fig. 3 Variation of the correctional factor Ψ_X^c with the transported quantities

where $\alpha_X = 0.2815$ and $\beta_X = -0.134$. The cost correctional factor Ψ_X^c variation is presented in figure Fig. 3.

Considering (8) the variables presented in Table 3 were defined.

4.2.3 Warehouses and disassembly centres' building sizes

The building size is established by the model, which will define the quantities of products to be transported from each warehouse to customer together with the amounts

Table 3 Economies of scale with transported quantities parameters

	X_m	Ψ_X^c
Forward flow—f		
Factory—Warehouse	X_{mij}^{f1}	$\Psi^c_{X_{mij}^{f1}}$
Warehouse—Customer	X_{mjk}^{f2}	$\Psi^c_{X_{mjk}^{f2}}$
Reverse Flow—r		
Customer—Disassembly centre	X_{mkl}^{r1}	$\Psi^c_{X_{mkl}^{r1}}$
Disassembly centre—Factory	X_{mli}^{r2}	$\Psi^c_{X_{mli}^{r2}}$

of products that will be returned to the disassembly centre by the customers.

Defining the capacity of a warehouse as Q_2^w and the capacity of a disassembly centre as Q_2^r, and the respective costs of opening these facilities as f_j^w and f_l^r. The costs of opening a base warehouse C_1^w or a disassembly centre C_1^r are always referred to the capacity of a base warehouse Q_1^w or base disassembly centre Q_1^r, respectively.

To make a more realist modelling, the costs of these facilities are a function of the referred quantities, but only after these have been rounded to the next multiple of the minimum capacity of the facility, Q_2^w in the case of warehouses and Q_2^r case disassembly centres. Using the model presented in (6) and $\ell = 0.8$ the following is considered.

The cost of opening warehouse w with a storage capacity of Q_2^w at location j is:

$$f_j^w = C_1^w \cdot \left(\frac{Q_2^w}{Q_1^w}\right)^{0,8} \tag{9}$$

The cost of opening disassembly centre r with a storage capacity of Q_2^r at location l is:

$$f_l^r = C_1^r \cdot \left(\frac{Q_2^r}{Q_1^r}\right)^{0,8} \tag{10}$$

4.3 Objective function

The objective function modelling the supply chain defined in Sect. 3, can be defined as:

$$\begin{aligned}\min\ F = & \sum_{i\in I} f_i^p \cdot Y_i^p + \sum_{j\in J} f_{jq}^w \cdot Y_{jq}^w + \sum_{l\in L} f_{lq}^r \cdot Y_{lq}^r \\ & + \sum_{m\in M}\sum_{i\in I}\sum_{j\in J} c_{mij}^{f1} \cdot \Psi_{t_{ij}}^c \cdot \Psi^c_{X_{mij}^{f1}} \cdot X_{mij}^{f1} \cdot t_{ij} \\ & + \sum_{m\in M}\sum_{j\in J}\sum_{k\in K} c_{mjk}^{f2} \cdot \Psi_{t_{jk}}^c \cdot \Psi^c_{X_{mjk}^{f2}} \cdot X_{mjk}^{f2} \cdot t_{jk} \\ & + \sum_{m\in M}\sum_{k\in K}\sum_{l\in L} c_{mkl}^{r1} \cdot \Psi_{t_{kl}}^c \cdot \Psi^c_{X_{mkl}^{r1}} \cdot X_{mkl}^{r1} \cdot t_{kl} \\ & + \sum_{m\in M}\sum_{l\in L}\sum_{i\in I_0} c_{mli}^{r2} \cdot \Psi_{t_{li}}^c \cdot \Psi^c_{X_{mli}^{r2}} \cdot X_{mli}^{r2} \cdot t_{li} \\ & + \sum_{m\in M}\sum_{k\in K} c_{mk}^{u} \cdot d_{mk} \cdot U_{mk} \\ & + \sum_{m\in M}\sum_{k\in K} c_{mk}^{w} \cdot r_{mk} \cdot W_{mk}.\end{aligned} \tag{11}$$

4.4 Constraints

The constraints considered are the following:

– All the demand of each customer is taken into account

$$\sum_{j\in J} X_{mjk}^{f2} + U_{mk} = d_{mk}, \quad \forall m \in M, \forall k \in K. \tag{12}$$

– All the return for each customer is taken into account

$$\sum_{l\in L} X_{mkl}^{r1} + W_{mk} = r_{mk}, \quad \forall m \in M, \forall k \in K. \tag{13}$$

– Balance between return and demand volumes

$$\sum_{m\in M}\sum_{k\in K}\sum_{l\in L} X_{mkl}^{r1} \le \sum_{m\in M}\sum_{j\in J}\sum_{k\in K} X_{mjk}^{f2}. \tag{14}$$

– Existence of a maximal disposal fraction

$$\gamma \cdot \left(\sum_{m\in M}\sum_{k\in K}\sum_{l\in L} X_{mkl}^{r1}\right) \le \sum_{m\in M}\sum_{l\in L} X_{ml0}^{r2}. \tag{15}$$

– Maximum capacity for factories in the forward and reverse chains

$$g_i^p Y_i^p \ge \sum_{m\in M}\sum_{j\in J} X_{mij}^{f1}, \quad \forall i \in I. \tag{16}$$

$$g_i^p Y_i^p \ge \sum_{m\in M}\sum_{l\in L} X_{mli}^{r2}, \quad \forall i \in I. \tag{17}$$

– Minimum capacity for factories in the forward and reverse chains

$$t_i^p Y_i^p \le \sum_{m\in M}\sum_{j\in J} X_{mij}^{f1}, \quad \forall i \in I. \tag{18}$$

$$t_i^p Y_i^p \le \sum_{m\in M}\sum_{l\in L} X_{mli}^{r2}, \quad \forall i \in I. \tag{19}$$

– Maximum capacity for warehouses

$$\sum_{m\in M}\sum_{i\in I} X_{mij}^{f1} \le g_{jq}^w Y_{jq}^w, \quad \forall j \in J. \tag{20}$$

– Minimum capacity for warehouses

$$\sum_{m\in M}\sum_{i\in I} X^{f1}_{mij} \geq t^{p}_{jq} Y^{p}_{jq}, \quad \forall j \in J. \tag{21}$$

– Maximum capacity for disassembly centres

$$\sum_{m\in M}\sum_{i\in I_0} X^{r2}_{mli} \leq g^{p}_{lq} Y^{p}_{lq}, \quad \forall l \in L. \tag{22}$$

– Minimum capacity for disassembly centres

$$\sum_{m\in M}\sum_{i\in I_0} X^{r2}_{mli} \geq t^{p}_{lq} Y^{p}_{lq}, \quad \forall l \in L. \tag{23}$$

– Warehouses and disassembly centres act as cross-docking platforms

$$\sum_{m\in M}\sum_{i\in I}\sum_{j\in J} X^{f1}_{mij} = \sum_{m\in M}\sum_{j\in J}\sum_{k\in K} X^{f2}_{mjk} \tag{24}$$

$$\sum_{m\in M}\sum_{k\in K}\sum_{l\in L} X^{r1}_{mkl} = \sum_{m\in M}\sum_{l\in L}\sum_{i\in I_0} X^{r2}_{mli} \tag{25}$$

– General constraints

$$U_{mk}, W_k, X^{f1}, X^{f2}, X^{r1}, X^{r2} \geq 0;$$
$$Y^{p}_{i}, Y^{w}_{jq}, Y^{r}_{lq} \in \{0, 1\}. \tag{26}$$

5 Optimization of closed-loop supply chains using ACO

The objective of this work is to design a CLSC by selecting the location of factories, warehouses (distribution centres) and disassembly centres from a set of pre-established possible locations to ensure that clients' demands are satisfied. This paper extends the ACO approach proposed in Esteves et al. 2012 to supply chain design considering nonlinear factors. The developed algorithm determines the production and storage amounts for the factories, warehouses and disassembly centres, as well as which facilities serve which products to specific clients. So as to minimize the nonlinear objective function (11).

The model considers that the building sizes are always multiples of the minimum allowed capacity for the type of facility being analysed, and it can be applied to any kind of supply chains: forward, reverse or closed loop. For example, if the total amount of products passing through warehouse *a* is 18,324 units and the minimal capacity is 5,000, the capacity Q_{2a} to be considered in the estimation of the cost associated to that warehouse is $Q_{2a} = 20{,}000$ units.

The next sections describe the main structures and processes of the algorithm, as well as its specific parametrization and dynamics.

5.1 General proposed algorithm

The algorithm was developed with the intent of being applicable to the design of any kind of supply chain, with forward and reverse flows, i.e. closed-loop flows. To achieve this purpose, it was developed for CLSCs, since it covers all the possible flows.

In an CLSC, the products are produced at the factories and distributed to customers through warehouses. When these products reach their end-of-life state, they are returned from customers and can either be sent to recycling or disposal at disposal facilities. This is done through a disassembly centre, which collects the returned products and sends them to the factories, in case of recycling, or to disposal.

A generic CLSC is presented in Fig. 4, which served as a modelling base. The developed algorithm is called SCAnt-NLDesign and has the structure of a generic ACO algorithm (Dorigo and Stützle 2004). The main characteristic of ACO presented in Algorithm 1—SCAnt-NLDesign is that the search of solutions is conditioned by previous information left in the system at each interaction. This previous information, in the scope of ACO, is called pheromone trails and mimics the behaviour of ants while searching for food.

Algorithm 1 SCAnt-NLDesign

```
Initialize forward parameters
Initialize reverse disposal parameters
Initialize reverse reprocessing parameters
for (I_n = 1 : I_cl × rps) do
  for (ant = 1 : N_a) do
    Reset ant path variables
    Reset tabu list variables
    Select facilities
    Compute transition probability
    Stage 1: Satisfy all custumers' demand
    Stage 2.1: Fulfil all disposals from custumers to factories
    Stage 2.2: Fulfil all remaining returns from custumers to factories
    Compute Trail Length
    solution ← Compare ant solutions
  end for
  beat ant solutions ← Apply stagnation control daemon
  Update pheromones (daemon choice)
end for
```

The process of constructing an ant solution proceeds as follows. At each iteration, a colony of ants is activated. Each ant, with the purpose of satisfying the customers' requirements (demand and return quantities), creates its own network of facilities (factories, warehouses and disassembly centres) by selecting those that can probably generate better solutions, conditioning in this way the search space. Using this network, the ant will select one of the orders (product type/customer) and satisfy it, affecting to it a set of facilities. When all the orders are satisfied, the set of all chosen facilities together with the flows is called a solution, which will

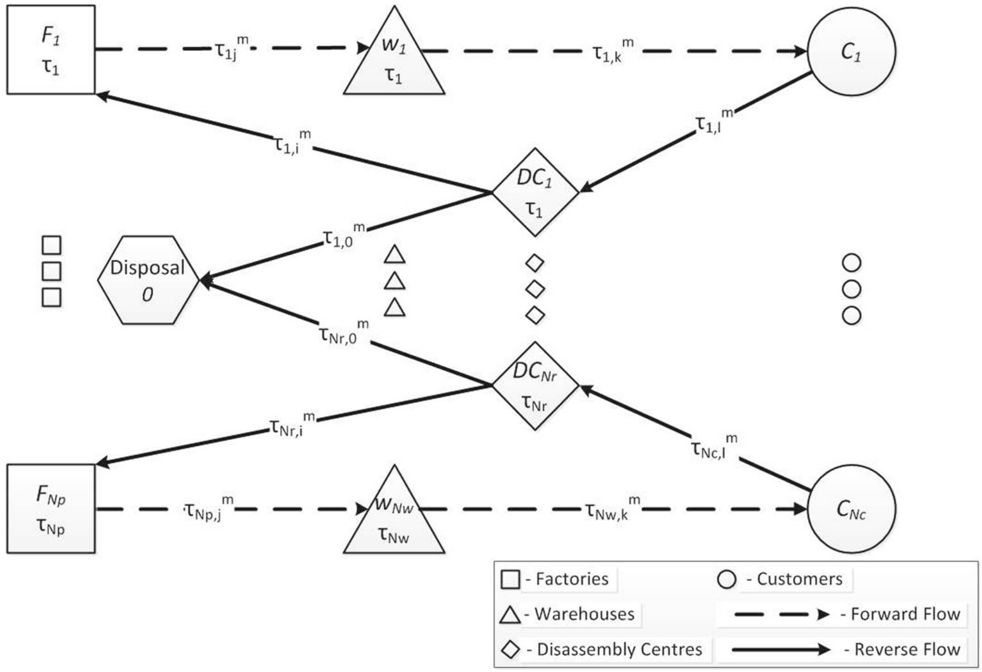

Fig. 4 Implementation base graph

be evaluated using the objective function presented in (11). When evaluating a solution, it has been considered that it may be economically more advantageous not to satisfy certain demand (or return), than to do so, incurring in a penalty cost. The model determines the economic advantage of delivering or not a certain quantity of product to a customer. For each product flow, the costs of non-satisfaction c_{mk} are compared with the total transportation costs of satisfying that order. If the non-satisfaction cost is lower than the cost of satisfying that same requirement, the adopted solution is that of non-satisfaction.

An iteration ends when all the ants of the colony have built their solutions, and will return the best solution found in that iteration. This is the solution with the lowest evaluated cost value, since the objective is to minimize the objective function.

To allow the conditioned search of the ants solutions, as prescribed by the ACO methodology, the idealized pheromone structure for SCAnt-NLDesign is represented in Fig. 4. In this figure, two kinds of pheromone deposits are used, one in the arcs and other in the nodes. The pheromones on the arcs condition the node selection made by an ant when, at a certain node, and are called connection-related pheromones. The pheromones on the nodes are called structural pheromones and condition the facilities selected by an ant to satisfy the customers' demands and returns.

Next sections describe Algorithm 1 in detail. First, the initialization steps of the SCAnt-NLDesign are presented. The main process of constructing ant solutions is described next. Follows the description on how the pheromones are updated, and finally the process of stagnation control daemon is detailed.

Table 4 Parameters

Parameter	Description
ρ	Pheromone evaporation rate
N_a	Number of ants per colony
α	Heuristic weight
β	Pheromone weight
rps	Repetitions
I_{cl}	Total number of cycle iterations
I_n	Iteration number n
Sz_{dl}	Size of a delivery
dl_{xp}	Number of expected deliveries

5.2 Initialization

In this step, general parameters and pheromone process parameters are defined, such as ρ, α, β and η. There will actually be four instances of the heuristic matrix, one for each echelon of the SC (as e.g. from factories to warehouses).

5.2.1 Parameters

Table 4 presents the parameters description and notation for the algorithm. The user defined parameters ρ, α and β are general characteristics of ACO algorithms as referred to in Dorigo and Stützle (2004). I_{cl} is the total number of iterations, that will condition the reset of the pheromone matrices (which will be addressed in the stagnation control daemon process in Sect. 5.5). The use of higher or lower values for I_{cl} implies a trade-off between computation time and solutions' space exploration. I_n is the number of the current iteration, being an iteration the construction of all the solutions by N_a ants that are part of the colony. The parameter rps is the number of times I_{cl} is repeated. The stopping criteria used in this algorithm are given by the multiplication of I_{cl} and rps.

In logistics, the size of the delivery Sz_{dl} is intrinsic to the transportation mode used. In this case, Sz_{dl} is the maximum number of units of any of the products that can be transported in a single delivery between entities in the supply chain network. Its value is defined by the user, and conditions the number of deliveries expected, dl_{xp}, for the forward flow:

$$dl_{xp} = \frac{d_{mk}}{Sz_{dl}}$$

and for the reverse flow,

$$dl_{xp} = \frac{r_{mk}}{Sz_{dl}}$$

5.2.2 Variables

The variables defined for the SCAnt-NLDesign algorithm are divided into sequential pheromone matrices (SPM), connection-related pheromone matrices (CPM), structural pheromone matrices, ant path variables, and tabu list variables. A summary of the pheromone matrices and heuristics used in SCAnt-NLDesign is given in Table 5. These variables are described in detail below.

(a) Sequential pheromone matrices (SPM)

These matrices are used to store the historical information regarding the sequence in which costumer's requirements (demands and returns) are satisfied.

There are four SPM matrices, two regarding the product sequences (one for the demand satisfaction and another for return satisfaction) and other two regarding the customers' sequencing. The SPM that refer to the product sequencing have a bi-dimensional structure, being the number of columns equal to the number of products present in the supply chain and number of rows the number of performed by the ants. The SPM for customers' demand and return satisfaction sequences are three-dimensional structures, where the number of rows is again the number of performed deliveries, the number of columns is the number of customers in the structure and its depth the number of products present in the supply chain.

Since there is not a specific allocation rule of customers' service to facilities (such as nearest customer, or other), SPM will store the information of previous solutions, which are used as a decision support structure of the customers' service sequence.

(b) Connection-related pheromone matrices (CPM)

Connection-related (or classic) pheromone matrices (CPM) are pheromone data structures analogous to those used in general ACO algorithms. Each entry in these matrices refers to a connection between nodes. There are four CPM, one for each echelon of the supply chain, see Table 5. These matrices have a tridimensional structure, where the number of rows and columns are the number of entities in question (e.g. the number of possible factories and the number of possible warehouses) and the depth is the number of products present in the supply chain.

(c) Structural pheromone matrices (STPM)

There is one STPM per type of facility (factories, warehouses, disassembly centres and disposal facility). In Table 5, τ_a where (a can be i, j or l) represent the probability that facility a will be chosen by an ant. STPM are used to select the facilities that will supply a given order to the customer. This process is explained in detail in Algorithm 1, where the construction of a solution will be detailed.

(d) Heuristics

Table 5 Variables

Description	Variable	Dimension
Sequential pheromone matrices (SPM)		
Product type demand Satisfaction sequence	$(\tau_m)^f$	$dl_{xp} \times N_v$
Customer demand Satisfaction sequence	$(\tau_k^m)^f$	$dl_{xp} \times N_c \times N_v$
Product type return Satisfaction sequence	$(\tau_m)^r$	$dl_{xp} \times N_v$
Customer return Satisfaction sequence	$(\tau_k^m)^r$	$dl_{xp} \times N_c \times N_v$
Connection pheromone matrices (CPM)		
Factory → Warehouse	τ_{ij}^m	$N_p \times N_w \times N_v$
Warehouse → Customer	τ_{jk}^m	$N_w \times N_c \times N_v$
Customer → Disassembly centre	τ_{kl}^m	$N_c \times N_r \times N_v$
Disassembly centre → Factory	τ_{li}^m	$N_r \times (N_i + 1) \times N_v$
Structural pheromone matrices (STPM)		
Factories	τ_i	$N_p \times 2$
Warehouse	τ_j	$N_w \times 2$
Disassembly centre	τ_l	$N_r \times 2$
Heuristics		
Factory → Warehouse	η_{ij}	$N_p \times N_w$
Warehouse → Customer	η_{jk}	$N_w \times N_c$
Customer → Disassembly centre	η_{kl}	$N_c \times N_r$
Disassembly centre → Factory	η_{li}	$N_r \times (N_i + 1)$

Table 6 Ant path variables

Variable	dl_{nf}	Dimension
A_p^f	$\{i, j, k, m, dl_q^n, dl\}$	$dl_{nr} \times \|dl_{nf}\|$
A_p^r	$\{i, l, k, m, dl_q^n, dl\}$	$dl_{nr} \times \|dl_{nf}\|$

The heuristics values referred in Table 5, represent previously known information about the problem and are typically inversely proportional to the distance between two facilities or customers' locations (Dorigo and Stützle 2004). Considering a and b as any two facilities η_{ab} is computed as:

$$\eta_{ab} = \frac{1}{t_{ab}} \tag{27}$$

where, t_{ab} is the distance between facilities a and b.

(e) Ant path variables

In Table 6, the matrices A_p^f and A_p^r are defined. These matrices are the ant paths in the forward and return chains, respectively, and record the decisions made by the ants. In both matrices the number of rows is the number of expected deliveries dl_{xp}. In each row is registered, for each delivery, the origin, destiny, product, transported quantity and used facilities. This information regarding each delivery will define the number of columns in each of the ant path matrices. It is also registered if the delivery is made or not, represented by the binary variable $dl = \{0, 1\}$ since the developed algorithm allows for the non-delivery (or return) of any percentage of demand (or return) if it proves to be an economically advantageous decision.

Considering dl_{nf} the delivery information, dl_n the delivery number, where $dl_{nr} = \{1, ..., dl_{xp}\}$ and dl_q^n the quantity of product m delivered (or returned) to customer k from factory i through warehouse j (or disassembly centre l).

An ant solution is composed of A_p^f and A_p^r,

(f) Tabu list variables

These variables, given in Table 7, are used to control and track the maximum production and storage limits of the facilities. When the limits have been reached, the facility or customer becomes part of these lists.

5.3 Constructing an ant solution

In SCAnt-NLDesign, ants incrementally build solutions for the optimization problem. as represented in Fig. 5.

An ant builds a solution by following Stage 1 (forward chain) and Stage 2 (reverse chain). Stage 1 satisfy (or not) custumers' demand. In the reverse stages, firstly all disposals from custumers are fulfilled (Stage 2.1) and afterwards the remaining returns from custumers to factories are fulfilled (Stage 2.2).

Table 7 Tabu list and demand control variables

	Description	Variable
Forward chain	Tabu factories	Γ_i^f
	Tabu warehouses	Γ_j
	Tabu customers	Γ_k^f
	Unsatisfied demand of product m for customer k	d_{mk}^u
	Satisfied demand of product m for customer k	d_{mk}^s
Reverse chain	Tabu factories	Γ_i^r
	Tabu disassembly centres	Γ_l
	Tabu customers	Γ_k^r
	Unsatisfied return of product m for customer k	r_{mk}^u
	Satisfied demand of product m for customer k	r_{mk}^s
	Unsatisfied disposal of product m for customer k	$r0_{mk}^u$
	Satisfied disposal of product m for customer k	$r0_{mk}^s$

At each stage, a generic algorithm is followed, which will be described in Algorithm 2. The termination conditions for the stages are given in Table 8.

A description of the processes select facilities, compute transition probability and compute trail length of Algorithm 1 and the processes select node and satisfy delivery lot size of Algorithm 2 are described below.

Algorithm 2 Generic stage to build an ant solution

```
while (Termination condition not satisfied) do
  Select node
  Satisfy delivery lot size
  Update facilities and products tabu lists
  Save path
end while
```

5.3.1 Select facilities

In this process, an ant chooses which facilities are used to construct a solution. The customer requirements should be fulfilled, i.e. the total demand and return should be satisfied considering the production and storage limits. This process selects the number of facilities using the STPM matrices and conditions the initial values of the tabu lists regarding facilities.

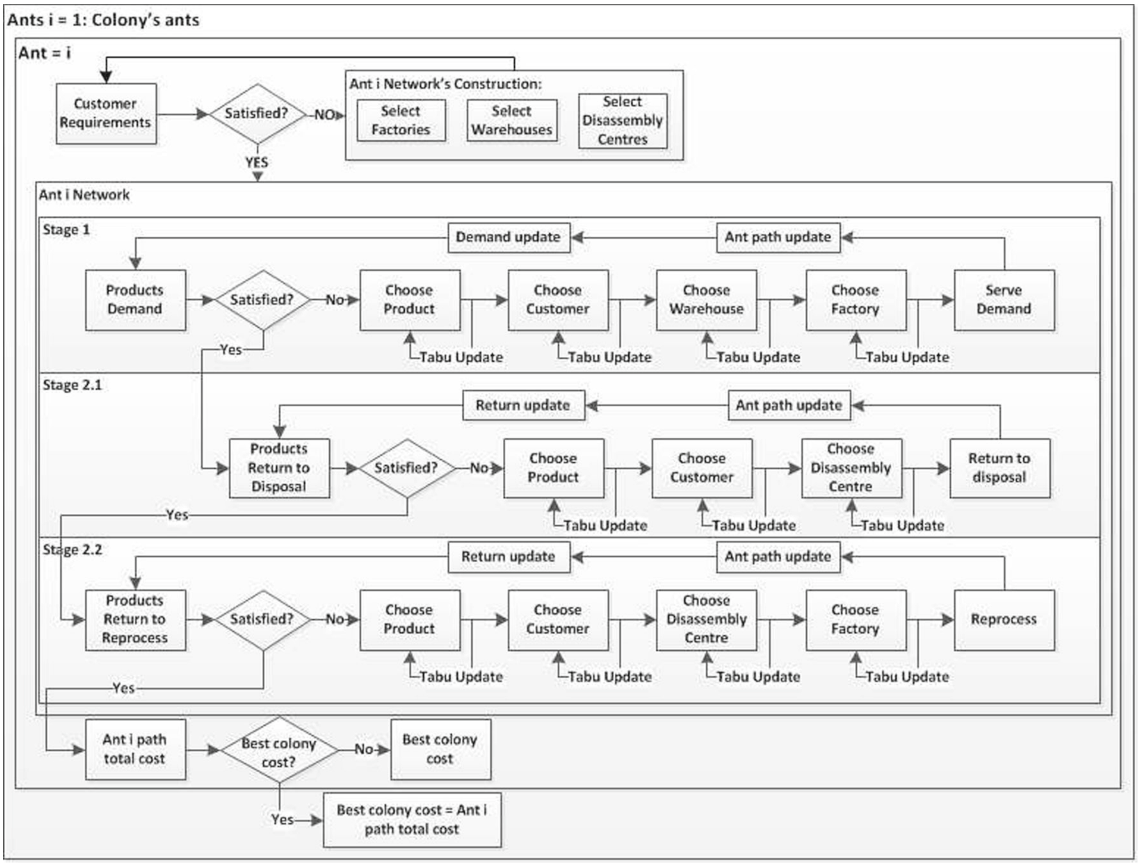

Fig. 5 Scheme for constructing an ant solution

5.3.2 Compute transition probability

In this step, the probability of an ant choosing a certain connection is computed. The transition probability between factories and warehouses is shown as an example:

$$p_{ij}^{m} = \frac{\left[\tau_{ij}^{m}\right]^{\alpha} \cdot \left[\eta_{ij}\right]^{\beta}}{\sum_{i \notin \Gamma_i^f} \sum_{j \notin \Gamma_j} \left[\tau_{ij}^{m}\right]^{\alpha} \cdot \left[\eta_{ij}\right]^{\beta}} \tag{28}$$

The p_{ij}^m matrix is tridimensional due to τ_{ij}^m, where the depth is determined by the number of products m. A similar approach is used to compute all other transition probabilities, namely the ones regarding warehouses and customers, customers and disassembly centres, and disassembly centres and factories. It must be noted that every time a facility enters the tabu list, all transition probabilities in which that facility is involved must be computed again, due to the denominator of the transition probability in (28).

5.3.3 Select node

In the node selection processes, the ant will choose the product it will deliver (or return), the customer to be served and the facilities it will use to satisfy that product order, conditioned by the tabu lists for products, customers and facilities, the previously built network (facilities selection in Sect. 5.3.1), the previously existing information stored in the pheromone matrices associated with products τ_m and customers τ_k^m and the transition probabilities previously determined (transition probability in Sect. 5.3.2) In the forward chain, the ant will choose which order will be served, from the product orders not yet satisfied. After choosing the order to be served, the ant selects the warehouse that will supply such order. Such selection is made taking into account the warehouse tabu list and the previously calculated client to warehouse transition probabilities. In this step, the variable that controls the number of times that warehouses are used is also updated. In the end, the ant will choose the factory that will produce those items taking into consideration both the factory tabu

Table 8 Termination conditions for the several stages

Stage	Termination condition
Stage 1	All customer demand satisfied
Stage 2.1	All disposals from customers to factories fulfilled
Stage 2.2	All remaining returns from customers to factories fulfilled

list and the previously calculated warehouse to factory transition probabilities. The variable that controls the number of times that factories are used is also updated. This sub-process is repeated until all customers' demand have been satisfied.

In the reverse process, the algorithm proceeds in the same way, but considering the reverse supply chain.

5.3.4 Satisfy delivery lot size

This step analyses the customers' requirements of the chosen product and compares it to both the remaining storage capacity of the facility (warehouse or disassembly centre) and the remaining production capacity of the selected factory. The smallest of these will determine the size of the delivered lot, so that no capacity restrictions are violated.

5.3.5 Compute trail length

This step evaluates the total cost of the solution designed by the ant. This process analyses the number of facilities, paths, transported quantities and products in each solution. The constructed solution is evaluated using the objective function in (11). The capacity of the warehouses and disassembly centres considered is the sum of the total amount of products passing through, rounded to the next multiple of each facility minimum capacity.

For example, if the total amount of products passing through warehouse a is 18,324 units and the minimal capacity is 5,000, the capacity Q_{2a} to be considered in the estimation of the cost associated to that warehouse is $Q_{2a} = 20{,}000$ units.

The cost of non-satisfaction of the demand and return of any customer is also considered in this step, by comparing the transportation cost of satisfaction with those of non-satisfaction. The cost that better reflects the objective of minimizing the costs, will be the one considered.

5.4 Pheromone update

This step chooses which solution is used to update the pheromone matrices, either the iteration-best or the best-so-far solution. This is achieved by gradually increasing the frequency in which the best-so-far tour is chosen for the trail update.

The following Eqs. (29) and (30) are used to compute the probability of choosing the best-so-far-ant solution (p_{bs}) and the iteration-best-ant solution (p_{ib}).

$$p_{bs} = 0.1 + 0.9 \cdot \frac{I_n}{I_{cl} \cdot rps} \tag{29}$$

$$p_{ib} = 1 - 0.1 \cdot \left(1 - \frac{I_n}{I_{cl} \cdot rps}\right) \tag{30}$$

Equations (29) and (30) show that the bigger the number of iterations, the bigger the probability of the best-so-far tour is chosen. This choice is made probabilistically according to a random uniform distribution, after normalization of both values. All pheromone matrices are subject to evaporation before any pheromone is laid on a path. The pheromone deposition is different for SPM, CPM and STPM. Next sections describe each pheromones update in more detail. All pheromone matrices use the following variables:

val	cost improvement capacity of A_p^f or A_p^r;
ρ	pheromone evaporation rate;
$\Delta\tau$	pheromone amount deposited;
I_n	current iteration;
$I_{(n-1)}$	previous iteration.

The pheromone update can be expressed by:

$$\tau(I_n) = \tau(I_{(n-1)}) \times \rho + \Delta\tau \tag{31}$$

5.4.1 Update of sequential pheromone matrices

The SPM regarding the product sequence pheromone matrices are updated A quantity $\Delta\tau_a$ corresponding to 10 % of val is added to the pheromone matrix in the product used in the delivery.

The pheromone matrices regarding sequences in which clients are served (or return products) are updated in a similar fashion, considering the product, and also the customer that was served in a certain delivery.

5.4.2 Update of connection-related pheromone matrices

First let us consider as an example the update of factory to warehouse pheromone matrix. Beyond the flow between a certain factory and a certain warehouse, it is also of interest to keep track of how many times a certain connection between these same facilities was used. If a connection had a certain flow Φ of product passing through it in only one delivery, and another connection had that same amount of product passing through it in two or more deliveries, than it is reasonable to consider that the second connection is of additional interest to more clients than the first. In accordance, the second connection should have a higher value of pheromone deposit. Thus, a connection that was used more often is of higher importance to more elements in the supply chain. Table 9 describes the CPM update variables.

The pheromone deposited in a connection depends on the flow passing through a connection (Φ_{ab}) and the number of times this connection was used (u_{ab}). Thus, the amount of pheromone to deposit in the connection from entity a to entity b is given by:

Table 9 CPM update variables

Description	Variable
Product flow matrices	
Factory → warehouse	Φ_{ij}
Warehouse → customer	Φ_{jk}
Customer → disassembly centre	Φ_{kl}
Disassembly centre → factory	Φ_{li}
Product use matrices	
Factory → warehouse	u_{ij}
Warehouse → customer	u_{jk}
Customer → disassembly centre	u_{kl}
Disassembly centre → factory	u_{li}

$$\Delta\tau_{ab} = \frac{\Phi_{ab}}{u_{ab}} \tag{32}$$

This pheromone deposit is a quantity inversely proportional to the total cost resultant from the ant path, I_{cl} and the forgetting factor (ρ).

5.4.3 Update of structural pheromone matrices

Structural pheromone matrices are updated with a quantity inversely proportional to the total cost resultant from the ant path, in a comparable fashion to what occurred with SPM. If a given factory, warehouse or disassembly centre is used by an ant, a pheromone quantity $\Delta\tau_a$ corresponding to 10 % of val is deposited.

5.5 Stagnation control daemon

This process was developed with the purpose of allowing the algorithm to escape from local optima. It keeps a record of the last few iterations and checks for stagnation behaviour. If such behaviour is detected (i.e. the best solution so far has not changed in a predetermined number of consecutive iterations) this process initiates a sequential re-initialization of the pheromone matrices, in the following order:

1. Sequential pheromone matrices;
2. Connection-related pheromone matrices;
3. Structural pheromone matrices.

The fact that pheromone matrices are sequentially re-initialized is due to the need of keeping some previous information, since re-initializing all the matrices at the same time would be the same as starting the optimization process from scratch.

This type of search space exploration methods is similar to the ones used by other ACO metaheuristics, such as MMAS (Dorigo and Stützle 2004). According to Dorigo and Stützle (2004), experience has shown that pheromone trail re-initialization, when combined with appropriate choices for the pheromone trail update, can be very useful to refocus the search on a different search space region. When such methods were applied, considerable improvement over using only iteration-best or best-so-far update were observed.

Several tests to the proposed SCAnt-NLDesign algorithm showed that the best results were obtained using the following condition to define the stagnation scenario:

– The objective function F defined in (11) does not decreases more than ϵ for $20 \times I_{cl}$ iterations (in this paper, ϵ is equal to 0.01).

6 Case study

6.1 Problem instance

The following case study consists of a CLSC where there are six possible locations for factories, ten possible locations for warehouses, ten possible locations for disassembly centres. Customers are considered as being grouped into fifty clusters located in the same geographical area, for simplicity these clusters will be referred to as customers. It is assumed that each costumer has a known demand for every type of product and will return a certain amount of every kind of product as "end-of-use product". The case study is also a multi-product model in which three different types of products are considered, which increases the problem complexity. The details of the case study are presented in Table 10.

The parameters identified in Table 4 were adjusted to achieve the more efficient convergence of the algorithm. The adjustment tests were done to get the best trade-off between the total cost F and the running time, by adjusting the values of I_{cl}, rps, and the number of ants in each colony N_a, for several runs of the algorithm.

The tests were executed for an $\alpha = 1$ and a $\beta = 2$, and was found that the more efficient convergence of the algorithm, was made for a number of 2 ants and a total of 400.000 iterations ($I_{cl} \cdot rps$) of the algorithm. The tests were run in a computer with a processor i7-2600 with 3.40 GHz and 8 Gb of ram. The algorithm was coded and ran in Matlab R2010b on a Windows 7 operating system.

6.2 Results

The algorithm ran 30 times using the parameters defined in the previous section. The obtained results are presented in Table 11.

The lowest total cost achieved was $54\,023 \times 10^3$ monetary units with an average running time of 11 h. All demand and return from customers was satisfied. For this lowest total

Table 10 Facilities and transportation data

Description	Parameter	Values		
Fixed cost per factory (m.u.)	f_i^p	9,000,000		
Cost (m.u.) per base warehouse ($Q1_j = 27.500\,un.$)	$C1_j$	1,000,000		
Cost (m.u.) per base disassembly centre ($Q1_l = 27.500\,un.$)	$C1_l$	1,000,000		
Disposal fraction	γ	0,1		
Capacity limits		Units		
Maximum factory production	g_i^p	50,000		
Minimum factory production	t_i^p	5,000		
Maximum warehouse capacity	g_i^p	50,000		
Minimum warehouse capacity	t_i^p	5,000		
Maximum disassembly centre capacity	g_i^p	50,000		
Minimum disassembly centre capacity	t_i^p	5,000		
Transportation cost per Km per product (m.u.)		Product type (m)		
		1	2	3
From factory to warehouse	c_{mij}^{f1}	5,72	4,93	4,78
From warehouse to customer	c_{mjk}^{f2}	7,30	6,94	5,93
From customer to disassembly centre	c_{mkl}^{r1}	7,19	5,48	5,95
From disassembly centre to factory	c_{mli}^{r2}	7,90	7,77	6,00

Table 11 Case study - Results

Min.	Total cost	
	Average	Std. deviation
($10^3 m.u.$)	($10^3 m.u.$)	($10^3 m.u.$)
54 023	54 297	212

cost were selected three factories, seven warehouses and four disassembly centres.

The factory to warehouse product flow and the disassembly centre to factory product flow are shown in Figs. 6 and 7, respectively. It can be observed, that in terms of production, there is a tendency to maximize the usage of factories 2 and 5, while factory 3 is used to absorb the rest of the demand. In the case of the reverse chain, the returned products are sent preferably to factory 3 and the factories 2 and 5 have about 1/3 of factory's 3 the usage. A sensitivity analysis was made and showed that, when the production capacity increases from 50.000 to 60.000 units, factory 3 is no longer used in the forward chain, but kept being a part of the reverse chain. A deeper analysis shows that the factories (forward and reverse) are always the same—2, 3, 5—including the disposal centre in the reverse chain, and the same is verified in the case of disassembly centres—3, 5, 9, 10. The used warehouses change, but warehouse 4 has always a smaller usage when compared to the others.

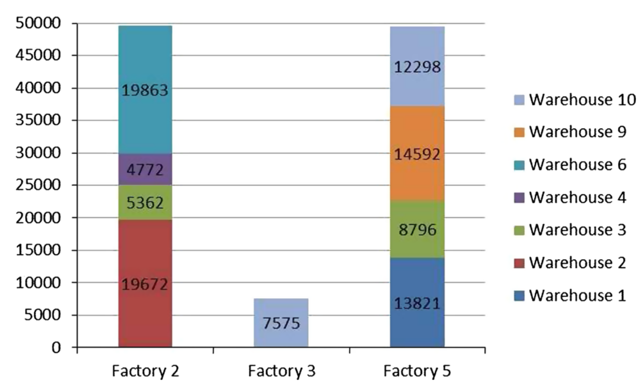

Fig. 6 Factory to warehouse product flow

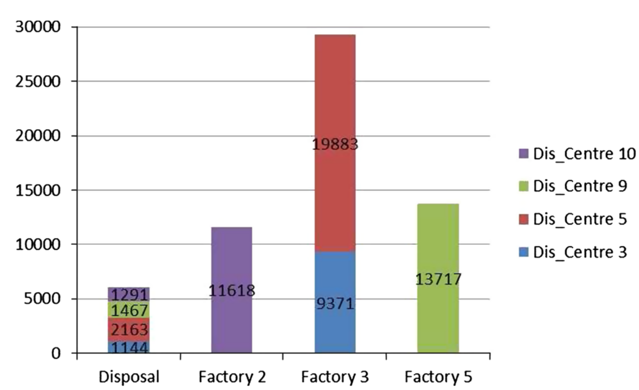

Fig. 7 Disassembly centre to factory product flow

Table 12 SCAnt-NLDesign and MILP results

Objective function terms	Model	Lowest cost (10^3) m.u.
Nonlinear	MILP	106 879
	SCAnt-NLDesign	54 023
Linear	MILP	84 842
	SCAnt-Design	85 127

6.3 Comparison between SCAnt-NLDesign and MILP results

The instance of a supply chain described here have also been optimized using a commercial MILP solver (GAMS/CPLEX). The results are presented in Table 12. Please note that the results for the linear case were already presented in Esteves et al. (2012). The nonlinear correctional factors introduced in this work were implemented in MILP using standard classical linearization methods, such as piece-wise linearization, which introduces a bias into the real solution. Further, the MILP approach became more complex and difficult to solve in GAMS/CPLEX, which was translated in a solution with a gap of 76.3 %.

On the other hand, the improvement achieved by SCAnt-NLDesign with the nonlinear model is very significant, being the cost reduced in 37 %. Note however that this reduction was expected due to the nonlinear terms introduced in the supply chain. Another significant achievement in the proposed SCAnt-NLDesign is the much better results it presents when compared with the MILP approach. It is clear the MILP cannot get close to the solution in the nonlinear case. The results obtained with a MILP approach are 98 % higher than using SCAnt-NLDesign which allows us to conclude that the proposed SCAnt-NLDesign algorithm is adequate for modelling supply chains with nonlinear dimensioning factors.

In terms of computational time, the SCAnt-NLDesign algorithm took in average 11h to reach a solution, about 40 % less time than the MILP model, which needed 17h30. Note that the SCAnt-NLDesign allows the use of any kind of terms, linear or nonlinear, without any significant increase in the computational time.

7 Conclusions

This paper introduces the effect of economies of scale in the design of CLSCs. Three economies of scale are considered: the tapering principle (scale economies with the distance), scale economies with transported quantities, and scale economies related to the warehouses and disassembly centres capacities. These economies of scale are nonlinear, and are expressed as correctional factors in the objective function. The objective is to make the model as realistic as possible.

To show this influence, instances of CLSCs without nonlinear factors (Salema et al. 2006; Esteves et al. 2012) were compared to the ones developed in this work. This comparison showed that the inclusion of nonlinearities in the model conducted to a much lower total cost (37 %) when compared to the previous linear model.

Future work can consider a multi-objective approach, where the minimization of environmental impacts and the costs are evaluated simultaneously. Further, uncertainty in the CLSC model can also be considered, as already addressed in other works (Klibi et al. 2010; Salema et al. 2010).

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