

A stochastic model for a multi-period multi-product closed loop supply chain

Susana Baptista^{*†}, Maria Isabel Gomes^{*}, Ana Paula Barbosa-Póvoa^{*‡}

^{*}*Centro de Matemática e Aplicações, FCT, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal,
E-mail:sbb@fct.unl.pt*

[†]*Centro de Matemática e Aplicações, FCT, Universidade Nova de Lisboa, 2829-516 Caparica, Portugal,
E-mail:mirg@fct.unl.pt*

[‡]*CEG-IST, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais, 1049-001
Lisboa, Portugal.
E-mail:apovoaa@ist.utl.pt*

Abstract

In this work we propose a stochastic model for the design and planning of closed-loop supply chains. Uncertainties in demand and return volumes are modelled together with uncertain transportation costs. A two-stage stochastic programming is developed and a sensitivity analysis to the worst-case probability is performed in order to test the solution robustness. Finally, in order to prove the goodness of the stochastic approach, the value of the stochastic solution and the value of perfect information are computed. An example based on a real case shows the model applicability.

Palavras chave: Closed-Loop Supply Chain, Design and Planning, Two-stage Stochastic Optimization.

1 Introduction

Nowadays the integration of forward and reverse flows in supply chains is a major concern for industries resulting in a high interest at the academia level. In such context, as points out [Fleischmann et al., 2001], the simultaneous design of the forward and reverse channels may lead to significant cost savings. Closed-loop chain network design has been an area of intensive research in the past decade as shown in the detailed literature review of [Akçali et al., 2009]. Authors have addressed strategic and tactical decisions for a single product in a single period [Fleischmann et al., 2001], for a single product in a multi-period setting [Beamon and Fernandes, 2004], for multi-products in a single period [Uster et al., 2007] and for multi-products in a multi-period setting [Salema et al., 2010]. Notice that in the aforementioned models, all the parameters are assumed to be deterministic. However, given the strategic nature of design decisions, several sources of uncertainty play a major role in the network behavior and therefore should not be disregarded. The stochastic closed-loop network design problem has been addressed by fewer authors when compared with the deterministic case. One of the first works was accomplished by [Inderfurth, 2005] for a single period in a multi-period setting and by [Listes, 2007] for a single product network in a single period. [Salema et al., 2007] and [Chouinard et al., 2008] proposed a two-stage stochastic model for a multi-product product network in a single period. [Pishvae and Rabbani, 2011] address uncertainties in the closed-loop supply chain through a robust optimization approach which avoids, according to the authors, the fitting of probability distributions for the uncertainty sources (transportation costs, demand and return volumes). [Zeballos, 2012] proposed a two-stage stochastic MILP model to study the quantity and quality of customer returned products. In this work, both sources are modelled simultaneously considering the maximization of total expected supply chain profit. Very recently, [Amin and Zhang, 2013] studied the design and planning of closed-loop supply chain in a three-step approach. Firstly, a qualitative approach is used to identify possible entities that will integrate the network design phase. Follows the evaluation of entities and the network configuration that in parallel approaches provide the inputs for the final step. In this third and last phase, a multi-objective MILP model selects and allocates customers' orders to the network entities. Uncertainties are addressed in phase two: a fuzzy approach models the uncertainty concerning the "importance" of entities to the network; a stochastic nonlinear MILP model tackles the network configuration where demand is the source of uncertainty. In this work different sources of uncertainties are tackled but not in an integrated way. [Cardoso et al., 2013] proposed a model for the design and planning of closed-loop supply chains and studied the impact that different network

configurations have on the Net Present Value (NPV). In addition, authors address the demand uncertainty in a multi-period setting. Demand uncertainty is modelled by a discrete probability distribution, which in a three time period planning horizon leads to a total of nine scenarios. A sensitivity analysis is performed on demand volumes and on the scenario probability distribution considering two different cases: the network flow structure is fixed or changes with the scenario. [Ramezani et al., 2013] proposed a multi-objective two-stage stochastic model where several sources of uncertainty such as selling prices, costs and demand and return rates are modelled. The ϵ -constraint methodology is used to approximate the Pareto Front considering three objectives: the maximization of profit and customer's service level and the minimization of defective parts acquired from the suppliers.

With two exceptions, all the above works addressed stochasticity in a single period context and in a single source of uncertainty. In this work we study three sources of uncertainties: transportation costs, demand and return volumes. The large fluctuations of diesel prices observed during the last 4 years: -21%, 15%, 19% and 5% (price increase in Portugal and when compared with the year before) and the fact that transportation costs represent a large fraction of the supply chain costs (in [Cardoso et al., 2013] transportation costs account on average for 38% of the total costs), turn the capturing of this uncertainty as one of the key features of the present model. Notice that since operational decisions are stochastic decisions, in the sense that they depend on the randomness realizations, and network design decisions are not stochastic, since they're to be taken in a unique way given all circumstances, a two-stage stochastic programming is adopted as a modelling framework. Further, random parameters are assumed to be discretely distributed with two possible realizations: normal case and worst case. Given the complexity of assessing a value for the worst case probability a sensitivity analysis to the worst-case probability is performed in order to test the solution robustness and support the network design decisions. Finally, in order to prove the goodness of the stochastic approach, the value of the stochastic solution and the value of perfect information are computed.

The paper is structured as follows. In the next section a detailed description and complete formulation of the model developed is presented. Section 3 is devoted to the computational tests and result analysis regarding a multi-period and multi-commodity network case study and finally, section 4 states the main conclusions.

2 The modelling approach

The model here proposed is an extension of the previous work of [Salema et al., 2010] in the sense that it addresses the uncertainty problem by considering transportation costs, customers' demands and customers' returns to be stochastic. Other than stochasticity, further refinements regarding the previous model include the fact that sales revenues are also considered, all the monetary values involved in future time periods are updated to their present value and that manufacturing and remanufacturing processes are now distinguishable in the sense that, beyond the recycled raw materials/recovered components that feed production, the model also accounts for the amount of brand-new materials/components to be acquired, so that the production costs are fully captured.

All entities composing the supply chain (Figure 1) act as product transformation points. Factories manufacture new products and/or remanufacture used ones. Warehouses execute postponement operations, which customize products to meet customers' demand. Customers' disposed products are collected by disassembly centres which, after sorting and disassembling operations, send components to be remanufactured or to be properly disposed.

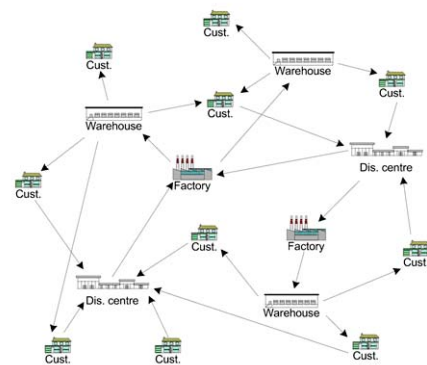


Figure 1: Closed-loop supply chain network structure.

Since operational decisions are to be undertaken for an entire time horizon, a two-unit scale is adopted for time modelling. Given a time horizon (e.g. 10 years), customers' demand has to be satisfied in some predefined time units, named the macro-period (e.g. yearly), while all planning decisions are to be taken in a smaller time scale, as months or weeks (the micro-period).

2.1 Stochastic modelling

In order to address stochasticity, we adopted a two-stage recourse model, where first stage decisions concern the design of the supply chain while second stage decisions regard the planning of the supply chain. We assumed that the random vector ξ that pieces together the stochastic components has finite support so that all the possible realizations of ξ are completely described by a set of scenarios. First stage decisions, involving the location of the four echelon network (plants, warehouses, customers and disassembly centres), are to be defined in order to maximize the expected net present value over all scenarios considered, and second stage decisions are to be defined for each one of the scenarios considered, so that different levels of production, storage and distribution flows will be obtained for each scenario.

As mentioned, three sources of uncertainty are modelled. Though a similar modelling approach is followed for each stochastic component, it is important to stress that each plays a different role in the model: demand is one of the models independent parameters, return uncertainties are variable constraint coefficients and transportation costs are objective function coefficients. As for the modelling, in particular for the demand modelling, if d denotes a product demand value and δ_s^d the demand variability factor for scenario s , the demand to be satisfied will be given by $d + \delta_s^d d = (1 + \delta_s^d)d$. Regarding the returned volume, since it is assumed that demand doesn't have to be totally satisfied, the return volumes will depend on the met demand. Thus, if x_d denotes the customer product demand volume that is satisfied, ρ the expected return rate and δ_s^r the return variability factor of scenario s , the total returned volume is given by $(1 + \delta_s^r)\rho x_d$. Finally for the modelling of transportations costs, where c denotes the transportation cost, x the transported amount and δ_s^t the transportation cost variability factor in scenario s , the total transportation cost will be given by $(1 + \delta_s^t)cx$.

2.2 Model formulation

Consider the sets and parameters defined in Appendix A and B and the following decision variables.

Continuous variables:

- $X_{spijt'}$ amount of product p dispatched from entity i to entity j at micro-period t' under scenario s
- $Y_{spit'}^{Sup}$ ($Y_{spit'}^{dc}$) amount of product p manufactured (remanufactured) by factory i at micro-period t' under scenario s
- $S_{spit'}$ amount of product p stocked at entity i at micro-period t' under scenario s
- U_{spit} unmet demand of product p in customer i at macro-period t under scenario s .

Binary variables:

Z_i equal to 1 if entity i is opened/included in the supply chain.

The problem can then be stated as:

Max F=

$$\sum_{s \in S} \sum_{t' \in T_m} \sum_{i \in I_{dc}} \sum_{j \in I_c} \sum_{p \in P(I_{dc})} \text{prob}_s \xi_p / (1 + Rt)^{1 + \text{Macro}P(t')} X_{spijt'} - \sum_{i \in I \setminus I_c} c_i^{fix} Z_i$$

$$\begin{aligned}
& - \sum_{s \in S} \sum_{t' \in T_m} \sum_{i \in I_f} \sum_{p \in P(I_f)} \text{prob}_s c_{pit'}^{pr1} / (1 + Rt)^{1 + \text{Macro}P(t')} Y_{spit'}^{Sup} \\
& - \sum_{s \in S} \sum_{t' \in T_m} \sum_{i \in I_f} \sum_{p \in P(I_f)} \text{prob}_s c_{pit'}^{pr2} / (1 + Rt)^{1 + \text{Macro}P(t')} Y_{spit'}^{dc} \\
& - \sum_{s \in S} \sum_{t' \in T_m} \sum_{i, j \in I, i \neq j} \sum_{p \in P(\text{Set}(i))} \text{prob}_s (1 + \delta_{s \text{Macro}P(t')}^t) c_{pit'}^{tr} / (1 + Rt)^{1 + \text{Macro}P(t')} d_{ij} X_{spij t'} \\
& - \sum_{s \in S} \sum_{t' \in T_m} \sum_{i \in I_{dc}} \sum_{p \in P(I_{dc})} \text{prob}_s (1 + \delta_{s \text{Macro}P(t')}^t) c_{pit'}^{tr} / (1 + Rt)^{1 + \text{Macro}P(t')} d_{i \text{Disp}} X_{spi \text{Dispt}'} \\
& - \sum_{s \in S} \sum_{t' \in T_m} \sum_{i \in I \setminus I_c} \sum_{p \in P(\text{Set}(i))} \text{prob}_s c_{pit'}^{st} / (1 + Rt)^{1 + \text{Macro}P(t')} S_{spit'} \\
& - \sum_{s \in S} \sum_{t \in T} \sum_{i \in I_c} \sum_{p \in P(I_{dc})} \text{prob}_s c_{pit'}^u / (1 + Rt)^{1+t} U_{spit} \tag{1}
\end{aligned}$$

s.t.

$$\sum_{\tilde{p} \in P_{I_{sup}}} \beta_{\tilde{p}p} X_{s\tilde{p}i(t' - \tau_{Sup,i})} = Y_{spi(t' + \phi_{\tilde{p}p}^{Sup})}^{Sup} \quad s \in S, p \in P(I_f), i \in I_f, t' \in T_m : (t' - \tau_{Sup,i}) \in T_m \tag{2}$$

$$\sum_{\tilde{p} \in P_{I_{dc}}} \beta_{\tilde{p}p} X_{s\tilde{p}i(t' - \tau_{dc,i})} = Y_{spi(t' + \phi_{\tilde{p}p}^{dc})}^{dc} \quad s \in S, p \in P(I_f), i \in I_f, t' \in T_m : (t' - \tau_{dc,i}) \in T_m \tag{3}$$

$$S_{spi(t'-1)} + Y_{spit'}^{Sup} + Y_{spit'}^{dc} = S_{spit'} + \sum_{j \in I_{dc}} X_{spij t'} \quad s \in S, p \in P(I_f), i \in I_f, t' \in T_m : (t' - 1) \in T_m \tag{4}$$

$$\begin{aligned}
S_{spi(t'-1)} + \sum_{j \in I_f} \sum_{\tilde{p} \in P_{I_f}} \beta_{\tilde{p}p} X_{s\tilde{p}ji(t' - \tau_{ji} - \phi_{\tilde{p}p})} &= S_{spit'} + \sum_{j \in I_f} X_{spij t'} \\
s \in S, p \in P(I_w), i \in I_w, t' \in T_m : (t' - 1) \in T_m &\tag{5}
\end{aligned}$$

$$\begin{aligned}
S_{spi(t'-1)} + \sum_{j \in I_c} \sum_{\tilde{p} \in P_{I_c}} \beta_{\tilde{p}p} X_{s\tilde{p}ji(t' - \tau_{ji} - \phi_{\tilde{p}p})} &= S_{spit'} + \sum_{j \in I_f} X_{spij t'} + X_{spi \text{Dispt}'} \\
s \in S, p \in P(I_{dc}), i \in I_{dc}, t' \in T_m : (t' - 1) \in T_m &\tag{6}
\end{aligned}$$

$$U_{spit} \geq Z_i (1 + \delta_{st}^d) d_{pit} - \sum_{j \in I_w} \sum_{t' \in \text{Micro}P(t)} X_{spji(t' - \tau_{ji})} \quad s \in S, p \in P(I_w), i \in I_c, t \in T \tag{7}$$

$$\begin{aligned}
\sum_{j \in I_w} \sum_{\tilde{p} \in P_{I_w}} (1 + \delta_{\tilde{p}p}^r) \rho_{\tilde{p}p} X_{s\tilde{p}ji(t' - \tau_{ji} - \phi_{\tilde{p}p})} &= \sum_{j \in I_{dc}} X_{spij t'} \\
s \in S, p \in P(I_c), i \in I_c, t' \in T_m : (t' - \tau_{ji} - \phi_{\tilde{p}p}) \in T_m &\tag{8}
\end{aligned}$$

$$\begin{aligned}
X_{spi \text{Dispt}'} &\leq (1 - \alpha_p) \sum_{j \in I_c} \sum_{\tilde{p} \in P_{I_c}} \beta_{\tilde{p}p} X_{s\tilde{p}ji(t' - \tau_{ji} - \phi_{\tilde{p}p})} \\
s \in S, p \in P(I_{dc}), i \in I_{dc}, t' \in T_m : (t' - \tau_{ji} - \phi_{\tilde{p}p}) \in T_m &\tag{9}
\end{aligned}$$

$$\sum_{p \in P(I_f)} (Y_{spit'}^{Sup} + Y_{spit'}^{dc}) \leq \text{MaxCap}_i \times Z_i \quad s \in S, i \in I_f, t' \in T_m \tag{10}$$

$$\sum_{p \in P(I_f)} (Y_{spit'}^{Sup} + Y_{spit'}^{dc}) \geq MinCap_i \times Z_i \quad s \in S, i \in I_f, t' \in T_m \quad (11)$$

$$\sum_{p \in P(Set(i))} X_{spijt'} \leq MaxF_{ij} \times Z_i \quad s \in S, i, j \in I : i \neq j, t' \in T_m \quad (12)$$

$$\sum_{p \in P(Set(i))} X_{spijt'} \geq MinF_{ij} \times Z_i \quad s \in S, i, j \in I : i \neq j, t' \in T_m \quad (13)$$

$$\sum_{p \in P(Set(i))} S_{spit'} \leq MaxSt_i \times Z_i \quad s \in S, i, j \in I \setminus I_c : i \neq j, t' \in T_m \quad (14)$$

$$\sum_{i \in I_w} Z_i \geq \sum_{i \in I_f} Z_i \quad (15)$$

$$X_{spijt'} \geq 0 \quad s \in S, p \in P(Set(i)), (i, j) \in A, t' \in T_m \quad (16)$$

$$Y_{spit'}^{Sup} \geq 0, Y_{spit'}^{dc} \geq 0 \quad s \in S, p \in P(I_f), i \in I_f, t' \in T_m \quad (17)$$

$$S_{spit'} \geq 0 \quad s \in S, p \in P(Set(i)), i \in I \setminus I_c, t' \in T_m \quad (18)$$

$$U_{spit} \geq 0 \quad s \in S, p \in P(I_c), i \in I_c, t' \in T_m \quad (19)$$

$$Z_i \in \{0, 1\} \quad i \in I \quad (20)$$

The objective function (1) expresses the total expected supply chain profit composed by: the expected revenue (first term), the opening fixed costs (second term), the expected productions costs for manufactured products (third term) and for remanufactured products (fourth term), expected transportation costs between all entities (fifth term), expected disposal costs (sixth term), expected stock costs (seventh term) and finally the expected penalty cost for not serving the demand of the included customers. All monetary values are reduced to their present value and fixed investment costs are equally divided over the assets useful life. Since the time horizon considered (e.g. five years) is assumed to be smaller than the assets useful life (e.g. fifteen years), the assets value at the end of the time horizon is not considered. Notice that a long asset useful life assumption is only appropriate if the addressed industry is not of short life-cycle products, otherwise a procedure similar to the one proposed by [Cardoso et al., 2013] must be performed. Equation (2) ensures that for every scenario, for each manufactured product p , all the components needed in the factories at time t' are transformed into p , so that p is available at time $t' + \Phi_{\tilde{p}p}^{Sup}$, where $\Phi_{\tilde{p}p}^{Sup}$ is the respective production time. Equation (3) establishes an equivalent result for remanufactured products. Equations (4), (5) and (6) are the balance equations at factories, warehouses and disassembly centres that ensure that the total flow dispatched from the entity (outbound flow) is equal to the level stock changes plus the flow that is dispatched to the entity (inbound flow). Notice that in equations (5) and (6), for a given entity the outbound flow of product p at time t' depends on the inbound flow at time $t' - \tau_{ji} - \Phi_{\tilde{p}p}$ where $\Phi_{\tilde{p}p}$ is the processing time of product \tilde{p} into p , and τ_{ji} is the shipping time between the origin and destination entities. A main feature of this model is the fact that product demands are considered to be stochastic. Since a recourse programming context was adopted, penalties for not meeting customer's demands were considered in the objective function. The amount of unmet demand is precisely defined by equation (7), which states that for every scenario and for every client, the unmet demand of product p is the difference between the demand of the product and the amount that was delivered to the client during the macro-period t . Notice that a product that reaches a client at time t' , must have been dispatched from the warehouse at time $t' - \tau_{ji}$, thus the amount of p delivered to customer i is given by $X_{spji(t' - \tau_{ji})}$ summed over all warehouses j and all micro-time periods t' that occur between the macro-time periods $[t - 1, t]$. Regarding the demand of p , $Z_i(1 + \delta_{st}^d)d_{pit}$ ensures that only demands of clients that belong to the network are considered. Since d_{pit} is the expected demand, δ_{st}^d is the demand variability factor (measured as a percentage of the expected demand and dependent of t) under scenario s . Another new feature of this model regards the stochasticity of product returns which is handled by equation (8). This equation is the balance equation at customers that ensures that for every scenario, customer and product returned from customers, the amount of product p shipped to all disassembly centers at time t' must be equal to the amount of products \tilde{p} that are returned as p . Notice that the amount of products \tilde{p} to be returned as p , is the amount of \tilde{p} dispatched from all the warehouses at time $t' - \tau_{ji} - \phi_{(\tilde{p}p)}$ (where the parameters are defined as before) times the return rate of product \tilde{p} as p which is given by $(1 + \delta_{\tilde{p}p}^r)\rho_{\tilde{p}p}$. Since $\rho_{\tilde{p}p}$ is the expected return rate of \tilde{p} as p , $\delta_{\tilde{p}p}^r$ is the return variability factor (measured as a percentage of the expected return rate) under scenario

s. Equation (9) assures that disassembly centres can only send to disposal less than a fraction of the returned products, in order to comply with the recovery targets set by legislation. This equation can be easily modified to cover the case where the disposal represents a third player (e.g. external recycling) so that $(1 - \alpha_p)$ is the fraction of products not suitable for remanufacturing. The maximum and minimum production capacities of factories are defined by equations (10) and (11) respectively, the maximum and minimum flow capacities are defined by equations (12) and (13) respectively and finally (14) sets the maximum stock levels at the different entities where products may be stored, i.e. factories, warehouses and disassembly centres. Constraint (15) ensures that the number of warehouses should be at least as large as the number of plants, so that the distribution structure is more decentralized in order to better encompass the transportation costs variations. Constraints (16) to (20) establish variables' domains.

3 Example details and results analysis

The computational tests were performed on a multi-period and multi-commodity network based on the work addressed by [Salema et al., 2010] where a glass supply chain network was studied. This network superstructure was defined with possible five plants, eight warehouses, 18 customers and eight disassembly centres. Three different products were considered in the flows plants-warehouses (F1 to F3), six for the flows warehouses-customers (A1 to A6), one for the flows customers-disassembly centres (R) and finally two for the closing loop flows disassembly centres-plants (C1 and C2) and one for the flows suppliers-plants (S). A three-year time horizon, with one year macro-time unit and three months micro-time unit was considered (parameters' values may be found in Appendix C). Regarding the number of scenarios, a normal case (NC) and a worst case (WC) scenario were identified.

As mentioned, transportation costs were defined as $(1 + \delta_{sMacroP(t')}^t)C_{pit}^{tr}$ and an annual variability factor of 5% and 10% increase was set for the normal and worst cases respectively. Customers' demands were assumed to increase annually 2% under a normal scenario and decrease 10% in the worst case. Customers' returns were defined as $(1 + \delta_{spp}^r)\rho_{pp}$ where the values of the parameters ρ_{pp} and δ_{spp}^r were set as in Table 1. δ_{spp}^r was assumed to have a null value under the normal scenario $\delta_{NC\bar{p}\bar{p}}^r$ so that no variability is incurred, while for the worst case scenario $\delta_{WC\bar{p}\bar{p}}^r$, the values were defined in order to ensure that fewer products are collected and that products with a larger return rate also present a smaller variability.

Table 1: Values for Return fraction and variability factor.

			A1	A2	A3	A4	A5	A6
Return fraction		$\rho_{\bar{p}\bar{p}}$	0.45	0.7	0.5	0.8	0.4	0.9
Variability factor	Normal Case	$\delta_{NC\bar{p}\bar{p}}^r$	0	0	0	0	0	0
	Worst Case	$\delta_{WC\bar{p}\bar{p}}^r$	0.2	0.1	0.2	0.05	0.2	0.05

The model was implemented in OPL using CPLEX 12.4 as solver. All the tests reported were conducted on a laptop with a 2.4 GHz Core i5 processor and all recourse problems were solved on average in 2079 seconds with computing times ranging from 1200 till 2871 CPU seconds. Recourse problems have 22066 constraints and 26505 variables from which 39 are binary. All monetary values presented were rescaled by a 1/1000 monetary units (m.u.) factor.

In order to analyse the solution robustness to the worst case scenario probability, the net present value and the number of entities were computed for probability values ranging from 0 to 100% following an increment of 20%. The results obtained for the expected value of perfect information (EVPI) and the value of the stochastic solution (VSS) were also analysed. Since the problem under analysis is a maximization problem, it is well known (see [Birge and Louveaux, 1997]) that the expected value of perfect information is the defined as $EVPI = WS - RP$ and the value of the stochastic solution as $VSS = RP - EMV$, where RP denotes the recourse program presented above, WS the wait-and-see problem and EMV the expected result of the mean problem. Thus, $RP = \min_x E_{\xi} z(x, \xi)$ will define the recourse problem, $WS = E_{\xi}(\min_x z(x, \xi))$ the wait-and-see problem and $EMV = E_{\xi}(z(\min_x z(x, E(\xi)), \xi))$ the expected result of the mean problem.

Figure 2 shows that, for whatever problem considered, the net present value decreases when the worst case probability increases and that globally, for worst case probabilities levels above 60% the supply chain system becomes unprofitable. Figure 3 exhibits the fact that, for the expected mean value problem, the network structure has two different configurations and that again the 60% worst case probability level is the turning point. Notice that the graphs of the number of plants and distribution centres are overlapped in figure 3. When analysing the NPV decrease trends (Fig. 3), it becomes clear that the WS problem solution exhibits a linear trend due to the fact that the probabilities changes only affect the objective function coefficients. As for the EMV, the NPV decrease follows from the network topology decrease depicted by figure 3. Regarding the recourse problem, two different decreasing rates trends can be identified. A steepest NPV decrease for probability values up to 60% with an average loss of 500 (m.u.×10³) per a 20% probability increase, while for probability values above 60%, the average loss is only of 200 (m.u.×10³) per a 20% probability increase. Such trend difference may be explained by the fact that the system topologies of probability levels up to 40% include 14 customers, while for probability levels beyond 60%, only 7 customers are considered (see Table 2). Notice that as theoretically expected the RP solution value is bounded above by the WS solution value and below by the EMV solution value.

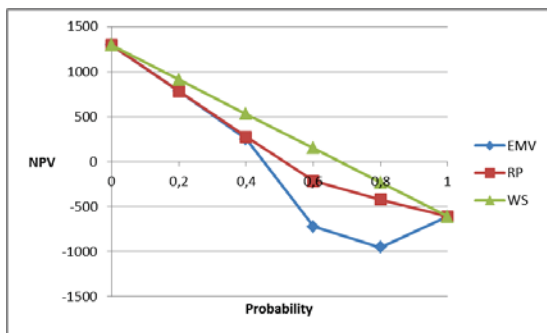


Figure 2: Relation between models' solutions and worst case probability.

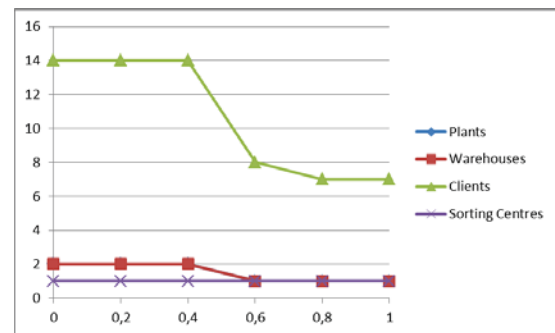


Figure 3: Relation between EMV network entities number and worst case probability.

Regarding the network structure solutions of the EMV and RP problems, Figure 3 and Table 2 show that as the worst case probability increases, the network becomes smaller, but in all cases the network design is quite robust to the worst case probability changes since only two different network configurations come up from the six probability levels analysed. As Table 3 points out no network modification occurs at probability levels up to 40%, at 60% the topology changes dramatically by reducing from two to one plant, from two warehouses to a single one, and cutting 7 of the 14 clients. Notice that for networks designed for worst case probability levels higher than 60% only small changes occur with a single customer being replaced by another one.

Table 2: Hamming distance between the EMV an RP solution.

Probability	0	0.2	0.4	0.6	0.8	1
Plants	0	0	0	0	0	0
Warehouses	0	0	0	0	0	0
Clients	0	0	0	1	0	0
Sorting Centres	0	0	0	0	0	0

Table 3: Hamming distance for RP solutions pairs.

Probability	(0,0.2)	(0.2,0.4)	(0.4,0.6)	(0.6,0.8)	(0.8,1)
Plants	0	0	1	0	0
Warehouses	0	0	1	0	0
Clients	0	0	7	1	1
Sorting Centres	0	0	0	0	0

Figure 4 presents the expected value of perfect information and the value of the stochastic solution. From Figure 2, it is clear that the deviation of the RP solution regarding the WS solution, and thus the EVPI value, increases for worst case probabilities up to 60% and decreases for probabilities that exceed that level, which explains the EVPI trend shown in Figure 4. Also the fact that the deviations of the EMV and RP solutions are larger for worst case probabilities ranging from 60 to 80% as it can be seen in Figure 2, explains the VSS trend depicted by Figure 4. Globally, both plots show that the 60% worst case probability level is the probability level for which it pays the most to access accurate information about the future (EVPI plot) and for which the cost of ignoring uncertainty is one of the largest (VSS plot). Another important feature is the fact that the value of the stochastic solution is almost insignificant for probability levels below 40%.

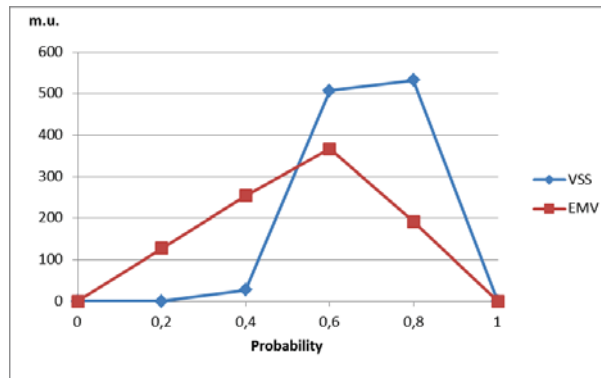


Figure 4: Relations between EVPI, VSS and worst case probability.

Finally, it is important to notice that, from a management perspective, the probability levels that are relevant are all that do not exceed the 60% level, since that is the range where the system is profitable. From a methodological perspective, any system analysis performed for probability levels below 40% can be achieved by applying the simple deterministic expected mean value problem, while for a worst case probability above that level, a stochastic programming approach should be adopted.

4 Conclusions

In this work we propose a two-stage MILP model for the design and planning of closed-loop supply chains accounting for different sources of uncertainty: product demands, return volumes, and transportation costs. Moreover, a multi-period and multi-product context is also contemplated.

The proposed formulation was applied to case based on a Portuguese glass company, where two scenarios (normal and worst case) were solved and results compared. A sensitive analysis was performed to assess the solution robustness regarding the worst case scenario probability. The expected value of perfect information and the value of the stochastic solution were also analysed. Results have shown that for values between 40% and 60% of the worst case probability, the network stops being profitable. It is between these same values that the network structure suffers the largest change. Half of the customers are no longer supplied and it can be observed a reduction to half on the number of opened factories and warehouses.

Given the fact that the range of the network profitability has been identified, a deeper analysis to the network robustness should be conducted. Such analysis will certainly involve a larger number of scenarios and thus the development of a decomposition strategy that will allow such modelling stands as future work. Furthermore, in this work we considered decisions to be taken in a risk neutral context, but in a context with a large uncertainty as it happens in the supply chain environment, risk management models should be encompassed. Thus, an extension of the present model that incorporates risk-averse measures such as the conditional value at risk will be undertaken. Finally, let us note that though we restricted the network design decisions to the first stage, a two-stage stochastic modelling approach would still be appropriate even if network decisions had to be taken throughout the time horizon, as long as those decisions had to be the same. On the contrary, if network decisions depended on the outcomes up to the moment where they're to be taken, then a multi-stage stochastic modelling approach should be

considered. Such extension will also be considered in the future so that the real multi-stage management decisions of the supply-chain system are fully captured.

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Appendix A:

The following **sets** were considered:

I_f, I_w, I_c, I_{dc} potential location for factories, warehouses, clients and disassembly centres, respectively

$$I = I_f \cup I_w \cup I_c \cup I_{dc}$$

$P_{(I_*)}$ products/components supplied by entities of set $I_* \subseteq I$

P_{Sup} components for manufactured products purchased at suppliers

S scenarios considered

T_m, T micro-time and macro-time periods, respectively

Set of all possible network flows:

$$A = \{(i, j) : \{i \in I_f \wedge j \in I_w\} \cup \{(i, j) : i \in I_w \wedge j \in I_c\} \cup \{(i, j) : i \in I_c \wedge j \in I_{dc}\} \cup \{(i, j) : i \in I_{dc} \wedge j = Disp\} \cup \{(i, j) : i \in I_{dc} \wedge j \in I_f\}$$

where $Disp$ is the disposal option made available at disassembly centres.

Appendix B:

The following **parameters** were considered:

Revenues

ξ_p sell price of product p

Costs

R_t interest rate

c_i^{fix} opening fixed cost of entity i

$c_{pit'}^{pr1}$ unit production cost of product p manufactured by factory i at time-period t'

$c_{pit'}^{pr2}$ unit production cost of product p manufactured by factory i at time-period t'

$c_{pit'}^{tr}$ unit transportation cost of product p supplied by entity i at time-period t'

$c_{pit'}^{st}$ unit cost of product p stocked at entity i at time-period t'

c_{spit}^u unit penalty cost of unmet demand of product p at customer i at macro-time t

Product parameters

$\beta_{\tilde{p}p}$ conversion product rate of component \tilde{p} into product p

$\phi_{\tilde{p}p}^{sup}$ manufacturing time of component \tilde{p} into product p

$\phi_{\tilde{p}p}^{dc}$ remanufacturing time of component \tilde{p} into product p

$\phi_{\tilde{p}p}$ processing time of product \tilde{p} into product p

$\rho_{\tilde{p}p}$ product \tilde{p} expected return rate as product p

d_{pit} demand of product p at customer i at macro-time t

α_p recovery target of product p

Time parameters

τ_{ji} transportation time between entities j and i

Distance parameter

d_{ji} distance between entities j and i

Scenario parameters

$prob_s$ scenario probability

$\delta_{sMacroP(t)}^t$ transportation cost variability factor at macro-time t for scenario s

δ_{st}^d demand variability factor at macro-time t for scenario s

$\delta_{s\tilde{p}p}^r$ product \tilde{p} return rate as product p variability factor for scenario s

Capacity parameters

$MaxCap_i$ maximum production capacity at factory i

$MinCap_i$ minimum production capacity at factory i

$MaxF_{ij}$ maximum flow between entities i and j

$MinF_{ij}$ minimum flow between entities i and j

$MaxSt_i$ maximum stock level at entity i

Functions of parameters

$Set(i) = I_*$ if $i \in I_*$

$MicroP(t) = \{t'_k, t'_{k+1}, \dots, t'_{k+n}\}$ set of micro-time periods between macro-time periods $(t-1)$ and t

$MacroP(t') = \lfloor \frac{t'-1}{n} \rfloor = t$ macro-time t at which micro-time t' belongs.

Appendix C:

$I_f = \{\text{Évora, Leiria, Lisboa, Porto, Setúbal}\}$

$I_w = \{\text{Braga, Coimbra, Leiria, Lisboa, Porto, Santarém, Setúbal, Viseu}\}$

$I_c = \{\text{Aveiro, Beja, Braga, Bragança, Castelo Branco, Coimbra, Évora, Faro, Guarda, Leiria Lisboa, Portalegre, Porto, Santarém, Setúbal, Viana Castelo, Vila Real, Viseu}\}$

$$\begin{aligned}I_{dc} &= \{\text{Braga, Coimbra, Leiria, Lisboa, Porto, Santarém, Setúbal, Viseu}\} \\P_{(I_f)} &= \{\text{F1, F2, F3}\} \quad P_{(I_w)} = \{\text{A1, A2, A3, A4, A5, A6}\} \\P_{(I_c)} &= \{\text{R}\} \quad P_{(I_{sc})} = \{\text{C1, C2}\} \quad P_{Sup} = \{\text{M1, M2}\} \\T_m &= 4 \quad T = 3 \\Rt &= 3\%\end{aligned}$$