Solving a multi-product, multi-depot vehicle routing problem by a hybrid method

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Abstract

This paper is focused on the delimitation of service areas and on vehicle routes definition in recyclable waste collection systems with more than one depot. Three types of materials have to be collected in separated routes, so the problem is modelled as a multi-product, multi-depot vehicle routing problem. A hybrid method is developed where a MIP solver is embedded inside a heuristic framework. The effectiveness of this method is tested by comparing the results obtained for some test instances when solved only by an exact formulation. The hybrid method is then applied to a medium size problem based on a real recyclable waste collection system.

Keywords: multi-depot, vehicle routing, multi-product, hybrid method, recyclable waste collection system.

1- Introduction

Recycling of packaging materials, imposed by the European Union, has forced member states to develop new collection systems. The traditional routes defined for organic waste do not fit the particularities where different vehicles, different collection rates, and different bin locations are required. These circumstances motivated the creation of two different waste collection systems: selective and undifferentiated. The selective collection is intended for recyclable products, while the undifferentiated is intended for non-recyclable products.

A recyclable waste collection system is responsible to collect, within a certain geographic area and on a regular basis, the three types of recyclable materials used in packaging (namely, paper, glass and plastic/metal) which are dropped by the final consumer into special containers. Afterwards, those materials are sorted at sorting stations and delivered to recyclers. Recyclable materials have different collection frequencies, so it is necessary to consider a planning horizon equal to the lowest collection frequency founded.

The present work aims to support tactical decisions for such systems, focusing on the delimitation of service areas of collection systems with more than one depot, and on vehicle routes definition. The vehicle routes definition is considered a tactical decision as the routes are to be maintain for a medium period of time (e.g. 6 to 12 months) due to the stability of the containers fulfilling rates. In this case, we are dealing with static routes and not with dynamic routes. Therefore, the routes defined are to be applied at the operational level, and only need to be revised due to seasonal demands or due to a significant increase in the quantities dropped by the population in the containers. Since we want to establish the service areas, the vehicle routes and we are dealing with three different products to be collected in separated routes, the problem is modelled as a multi-product, multi-depot vehicle routing problem. The problem is solved through a hybrid method where a MIP solver is embedded within a heuristic framework. The effectiveness of this method is tested by comparing the results obtained for some test instances solved only by an exact formulation. The hybrid method is then applied to a medium size problem based on a real recyclable collection system.

This paper is structured as follows. After a brief review of the literature on multi-depot vehicle routing problem (MDVRP) in Section 2, we describe the hybrid method and test its effectiveness in Section 3. The computational results obtained for a medium size problem are presented in Section 4. Finally, we draw conclusions and discuss future work directions.

2- Literature Review

MDVRP is usually defined by a graph $G=(V,A)$, where $V=\{1,\ldots,N+W\}$ is the vertex set and $A=\{(i,j):i,j\in V,i\neq j\}$ is the edge set. The vertex V is partitioned into two subsets $V_c=\{1,\ldots,N\}$ and $V_d=\{N+1,\ldots,N+W\}$, representing the set of cities or customers and the set of depots, respectively. At each depot are based $k\in\{1,\ldots,K\}$ vehicles of capacity Q_k. Each vertex i in V_c has a nonnegative demand p_i and a nonnegative service duration t_i. A distance matrix D = (d_{ij}) is associated with set *A*.

The MDVRP consists of building a set of vehicle routes in such a way that: (1) each route starts and ends at the same depot, (2) each customer is visited exactly once by a vehicle, (3) the total demand of each route does not exceed the vehicle capacity, (4) the total duration of each route (including travel and service times) does not exceed a preset limit so that (5) the total routing cost is minimized.

For the MDVRP, there are several models developed (exact and approximate approaches). Due to its NP-hard combinatorial nature, the models proposed in the literature are mostly heuristics-based. There are still few exact algorithms in the literature. Laporte et al. (1984), as well as Laporte et al.(1988), developed exact branch and bound algorithms for solving the symmetric and asymmetric version of the MDVRP, respectively. Recently, Baldacci and Mingozzi (2009) developed an exact method for solving the Heterogeneous Vehicle Routing Problem (HVRP) that is capable to solve, among other problems, the MDVRP. This algorithm is based on the set partitioning formulation, where a procedure is applied to generate routes and three bounding procedures are used to reduce the number of variables in the formulation. However, when analysing heuristic algorithms to solve MDVRP several ones have been proposed (Tillman and Cain (1972), Golden, Magnanti and Nguyen (1977), Renaud et al.(1996), Salhi and Sari (1997), Lim and Wang (2005), Crevier et al. (2007), among others).

Therefore we can conclude that few exact models for the multi-depot problems have been proposed, while many heuristic procedures exist for the same problem. The combination of these two methods has also received little attention from the academia. Therefore, this work explores this opportunity and proposes a hybrid method combining exact formulation with heuristic procedures to solve the multi-product, multi-depot vehicle routing problem.

3- Hybrid Method to Solve the Multi-Product, Multi-Depot VRP

3.1- Description

The main idea of this hybrid method is to relax the original model (Multi-Product, Multi-Depot VRP), and then set decision variables that present feasible values along the procedure.

The first step is then to relax the original Multi-Product, Multi-Depot VRP problem where more than one product is considered and the vehicles are restricted to start and finish at the same depot. This relaxation leads to the Single-Product, Multi-Depot VRP with Multi-Depot Routes, where we have just one product and multi-depot routes are allowed. By solving the first step of the model, we will have some collection sites that belong to a route that start and finish at the same depot and some collection sites that do not. For the “feasible” collection sites we set their assignment to the start and finish depot but not to a particular route or vehicle. After setting these variables the next step of the procedure is to run the Single-Product, Multi-Depot VRP, where all the routes are required to start and end at the same depot.

The first two steps of the algorithm are run for each of the products in study.

After the model Single-Product, Multi-Depot VRP has been run for all the products individually, in the solutions there are some collection sites that belong to routes of the same depot. For those sites, we set their assignment to that depot and then run the Multi-Product, Multi-Depot VRP (which has now the constraint that guarantees that the service areas are defined by depot). This is required because in the original problem, the “Multi-Product” problem, all types of products in each collection site have to be collected from the same depot. In Figure 1 it can be seen a schematic diagram of the hybrid method proposed where the example for three products (paper, glass and plastic/metal) is used.

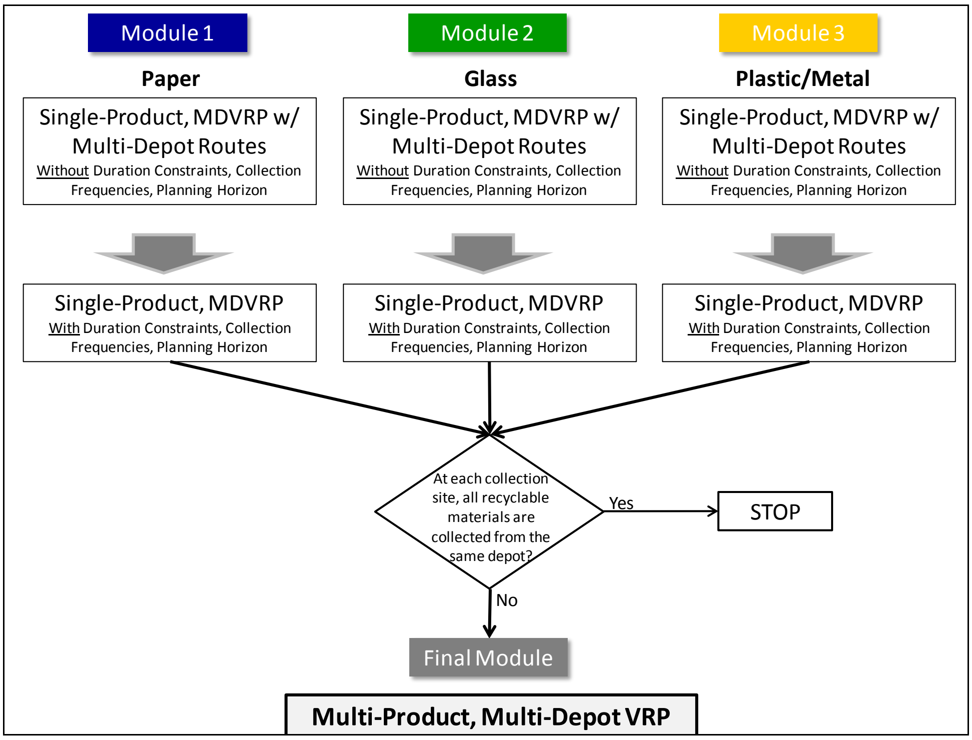

Figure 1 – Hybrid Method Modules

The method proposed is called "hybrid" since it combines exact formulations with the procedure that set the value of some variables. This algorithm solves three mathematical models: 1) Single-Product, Multi-Depot VRP with Multi-Depot Routes, 2) Single-Product, Multi-Depot VRP, 3) Multi-Product, Multi-Depot VRP. The last one is the original problem that has some fixed variables from the results of the first two models (which are relaxations of the original one). All mathematical formulations are based on the two-commodity flow formulation proposed by Baldacci et al. (2004) for the capacitated VRP (CVRP). We adapted Baldacci et al. (2004) formulation to comprise multiple depots and multiple products by adding a binary decision variable z_{ik} that links each collection site i to a vehicle route k, by adding the index m and k to the decision variables x and y, and by adding some constraints. This formulation requires the extend graph $\overline{G} = (\overline{V}, \overline{A})$ obtained from G by adding the vertex set V_f = $\{N+W+1, \ldots, N+2W\}$ which is a replica of the set of depots. Thus, $\overline{V} = V \cup V_f$, $\overline{A} = A \cup \{(i, j) : i \in V_c, j \in V_f\}$ and $d_{i(j+W)} = d_{ji}, i \in V_c, j \in V_d$.

All routes start at one real depot (set V_d) and end in the corresponding replica depot (set V_f). Each route is defined by two flow paths: one path from the real depot to the replica depot, defined by variables y_{ijmk} (representing the vehicle load, which increases along the route since we are collecting waste); the other path, the reverse one, starts at the replica depot and ends at the real depot, and is defined by variables y_{jimk} (representing the empty space on the vehicle, which decreases along the route). Besides the decision variables y_{ijmk} and y_{jimk}, this formulation has a binary variable x_{ijmk} which represents the routing solution (1, if site *j* is visited immediately after site *i*, to collect material *m*, by vehicle route *k*; 0, otherwise).

The objective function of the model focuses on minimizing the total distance travelled to collect all recyclable materials at collection sites over the timeframe.

3.2- Effectiveness Testing

In this section, we evaluated the effectiveness of the proposed hybrid method by applying it to a set of test instances and comparing some performance measures (i.e., lower bound, quality of the final solution and computational time) with the optimal solutions from the exact formulation. Due to the complexity of the multi-product, multi-depot vehicle routing problem, the instances that can be solved to optimality will typically be small. Therefore, eight small instances were generated since in the literature there are not available test instances for multi-product problems. The structure of these instances is presented in Table 1.

Table 1- Structure of the Test Instances

Instance	*Nº of Depots*	*Nº of Collection Sites*	*Nº of Recyclable Materials*	*Nº of Vehicles*
1	2	8	3	4
2	2	11	3	4
3	2	11	3	6
4	2	13	3	6
5	2	15	3	4
6	2	16	3	4
7	2	16	3	4
8	2	18	3	4

All the instances are solved by the branch-and-bound algorithm using GAMS/CPLEX Optimizer 12.1.0. An Intel(R) Core(TM) i7 CPU 930 @ 2,80 GHz is used.

The results obtained by applying the branch-and-bound and the hybrid method to the test instances are presented in Table 2.

Table 2 - Comparisons Between the Optimal Value and the Hybrid Method Solution

Instance	Exact Formulation			Hybrid Method			Deviation		
	Lower Bound	Opt. Value	CPU Time (sec)	Lower Bound	Opt. Value	CPU Time (sec)	Lower Bound	Opt.Value	CPU Time
1	589.3	691.1	10	613.7	691.1	2	4.1%	0%	-80%
2	570.1	669.4	104	585.1	682.6	43	2.6%	2%	-59%
3	770.8	872.7	232	788.0	872.7	40	2.2%	0%	-83%
4	802.1	875.1	6115	810.8	875.1	859	1.1%	0%	-86%
5	557.5	656.5	5520	567.2	656.5	487	1.7%	0%	-91%
6	759.2	904.3	987	770.0	904.3	648	1.4%	0%	-34%
7	660.3	750.4	5071	660.3	750.4	726	0%	0%	-86%
8	769.1	946.8	48917	779.2	946.8	2100	1.3%	0%	-96%

Deviation= (Hybrid Method Value/Exact Formulation Value – 1)×100

As mentioned above, three performance measures are compared to assess the effectiveness of the hybrid method: the root node lower bound, the solution quality and the CPU time to compute and prove the optimal solution. We calculate a percentage deviation between the values founded by the exact formulation and by the hybrid method for each of the performance indicators. From Table 2, we can see that, with the exception of instance 7, the root node lower bound provided by the hybrid method is of better quality than the one provided by the exact formulation without compromising the search for the optimal solution (which could happen in other examples). Again, with one exception (instance 2), the hybrid method always finds the optimal solution. For instance 2, the hybrid method finds a solution 2% worse than the optimal one. In terms of computational time, the hybrid method reaches the optimal solution in less time than the exact formulation.

These results show that the hybrid method is an effective method to produce a good or even the optimal solution in less time that the exact formulation.

4- Application to a medium size problem

In this section, the method developed is applied to a medium size problem, with 50 nodes. The nodes location is represented in Figure 2.

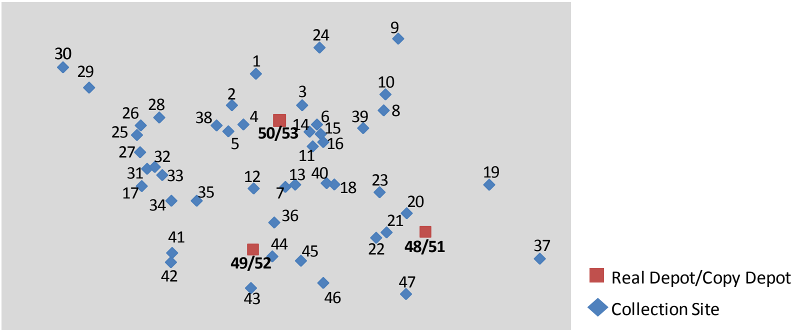

Figure 2 – Location of the Collection Sites and Depots

Since this problem is based on a real case study regarding a recyclable waste collection system, we have three recyclable materials to be considered (Paper, Glass and Plastic/Metal). The recyclable materials have different collection frequencies, so we considered a two-weeks planning horizon, which corresponds to 10 working days. Paper has to be collected four times over the timeframe, Glass has to be collected once and Plastic/Metal twice.

In order to illustrate how the hybrid method works, the detailed results obtained for each module are presented below. As mentioned above, all modules are solved by GAMS/CPLEX Optimizer 12.1.0, being computation time limited to 2 hour for each module.

"Paper" is the first recyclable material for which routes are design by the Single Product, Multi-Depot VRP with Multi-Depot Routes. The results provide five routes, where two are feasible (since they start and end at the same depot) and the other three are infeasible (see Figure 3). Therefore, for sites that are collected by feasible routes their assignment to a depot is fixed. The remaining sites are kept free when the Single-Product, MDVRP is runned. The solution obtained for the Single-Product, MDVRP involves two routes by depot, and the corresponding service areas are showed in Figure 3.

Single-Product, MDVRP with Multi-Depot Routes:

Routes	Feasible for MDVRP?
48-19-37-47-21-22-46-45-44-52	Infeasible
48-20-23-18-40-15-16-14-53	Infeasible
49-34-17-33-32-31-27-26-29-30-28-38-53	Infeasible
49-36-13-7-12-35-41-42-43-52	Feasible
50-3-39-8-10-24-1-2-5-4-53	Feasible

Single-Product, MDVRP:

Routes:

Depot 48/51:
48-19-37-47-51
48-21-22-40-18-23-20-51

Depot 49/52:
49-42-41-17-31-32-33-34-35-12-7-13-36-52
49-43-46-45-44-52

Depot 50/53:
50-1-2-28-30-29-26-27-38-5-4-53
50-14-16-15-39-8-10-24-3-53

Service Areas:

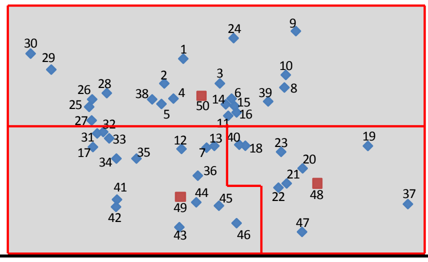

Figure 3 - Results Obtained for Module 1 (Paper)

For material "Glass", three infeasible routes and four feasible routes are the result obtained for the Single-Product, MDVRP with Multi-Depot Routes (Figure 4). When Single-Product, MDVRP is run, one route is defined to depot 48/51, two routes are defined to depot 49/52 and four routes are defined to depot 50/53. The resulting service areas have a different configuration when compared with the service areas defined by the collection of material "Paper".

Single-Product, MDVRP with Multi-Depot Routes:

Routes	Feasible for MDVRP?
49-36-13-7-12-35-34-17-31-32-38-53	Infeasible
49-41-42-43-52	Feasible
49-44-45-46-47-37-19-51	Infeasible
50-1-2-5-4-53	Feasible
50-14-15-40-18-23-22-21-20-51	Infeasible
50-26-25-29-30-28-53	Feasible
50-3-24-9-10-39-16-6-53	Feasible

Single-Product, MDVRP:

Routes:

Depot 48/51:
48-19-37-47-22-21-23-20-51

Depot 49/52:
49-35-34-17-31-32-12-36-44-52
49-45-46-43-42-41-52

Depot 50/53:
50-4-5-38-2-53
50-7-13-40-18-15-16-14-6-3-53
50-26-25-29-30-28-53
50-39-10-9-24-1-53

Service Areas:

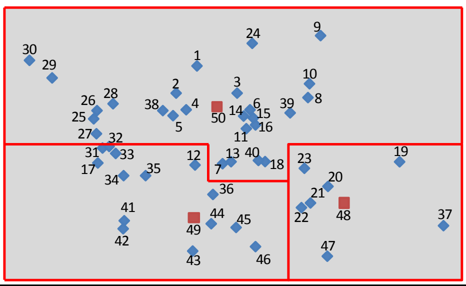

Figure 4 - Results Obtained for Module 2 (Glass)

For material "Plastic/Metal", the Single-Product MDVRP produces only two infeasible routes out of seven routes. When Single-Product, MDVRP is runned, one route is defined to depot 48/51, two routes are defined to depot 49/52 and four routes are defined to depot 50/53. The resulting service areas have a different configuration when compared to the service areas to collect material “Paper” and “Glass” (Figure 5).

Single-Product, MDVRP with Multi-Depot Routes:

Routes	Feasible for MDVRP?
48-20-23-21-22-47-37-19-51	Feasible
49-35-34-41-42-52	Feasible
49-36-7-13-40-18-15-11-53	Infeasible
49-43-45-44-52	Feasible
50-2-26-29-30-28-1-53	Feasible
50-3-9-10-8-39-6-14-53	Feasible
50-4-38-27-31-32-33-17-52	Infeasible

Single-Product, MDVRP:

Routes:

Depot 48/51:
48-20-23-21-22-47-37-19-51

Depot 49/52:
49-42-41-34-35-36-52
49-43-45-44-52

Depot 50/53:
50-1-28-30-29-26-2-53
50-3-9-10-8-39-6-14-53
50-11-15-18-40-13-7-53
50-33-17-31-32-27-38-4-53

Service Areas:

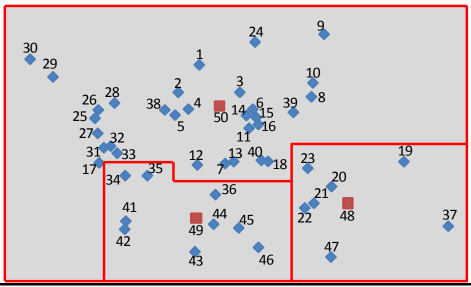

Figure 5 - Results Obtained for Module 3 (Plastic/Metal)

Since the resulting service areas are different among the three recyclable materials, is necessary to run the final module of the hybrid method – the Multi-Product, MDVRP. The collection sites with an infeasible assignment regarding service areas definition are identified – collection sites 7, 12 13, 17, 18, 31, 32, 33 and 40 have not all recyclable materials collected from the same depot. For example, collection site 40 is collected from depot 48 in Paper collection routes, and from depot 50 in Glass and Plastic/Metal collection routes.

The total distance travelled over the timeframe to collect all recyclable materials, where all routes start and end at the same depot and service areas are defined by depot is 2.523 Km. The vehicle routes for each depot are presented on Figure 6 and the corresponding service areas are shown in Figure 7. In this case, the final service areas configuration is equal to the service area configuration provided by module 1. However, this is not always the case in other examples.

Depot 48/51	Depot 49/52	Depot 50/53
Paper 48-19-37-47-51 48-20-23-18-40-22-21-51	Paper 49-36-13-7-12-35-34-33-32-31-17-41-42-52 49-44-45-46-43-52	Paper 50-4-5-38-27-26-29-30-28-2-1-53 50-14-16-15-39-8-10-24-3-5
Glass 48-19-37-47-21-22-40-18-23-20-51	Glass 49-35-34-17-31-32-12-7-13-36-44-52 49-45-46-43-42-41-52	Glass 50-2-38-5-4-53 50-14-15-16-6-3-53 50-26-25-29-30-28-53 50-39-10-9-24-1-53
Plastic/Metal 48-19-37-47-51 48-21-22-40-18-23-20-51	Plastic/Metal 49-35-33-32-31-17-34-52 49-36-7-13-45-44-52 49-43-42-41-52	Plastic/Metal 50-3-9-10-8-39-6-53 50-4-38-1-14-15-11-53 50-28-30-29-26-27-2-53

Figure 6 - Final Routes by Depot

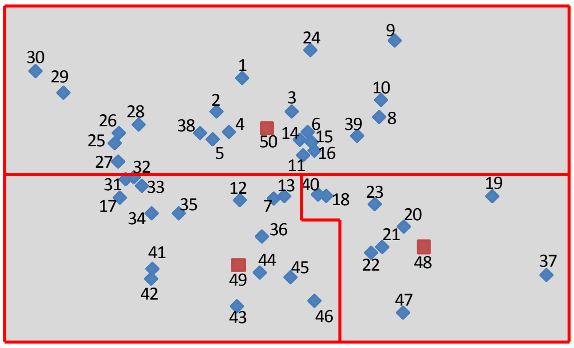

Figure 7 - Final Service Areas

The computational results for the hybrid method are presented in Table 3. We compare the performance of the hybrid method with the performance of the exact formulation for this problem. Within the same CPU time, the hybrid method has found a better solution than the exact formulation for the original problem (2.523 *vs.* 2.638). If the original problem is used to find the same solution provided by the hybrid method, the exact formulation needs 70 hours of CPU time, presenting a gap of 9,8%.

Table 3 – Computational Results Obtained for Hybrid Method and Exact Formulation

Methods	Opt. Value	GAP	CPU (hours)
Hybrid Method			
Module 1 (Paper)			
• Single-Product, MDVRP w/ Multi-Depot Routes	332	0%	0,5
• Single-Product, MDVRP	1350	6,6%	1
Module 2 (Glass)			
• Single-Product, MDVRP w/ Multi-Depot Routes	378	2,9%	1
• Single-Product, MDVRP	388	7,5%	1
Module 3 (Plastic/metal)			
• Single-Product, MDVRP w/ Multi-Depot Routes	366	2,6%	1
• Single-Product, MDVRP	747	5,7%	1
Final Module			
• Multi-Product, MDVRP	2.523	8,9%	2
Exact formulation for the Multi-Product, MDVRP	2.638	14,1%	7,5

5- Conclusions

The present work addresses the multi-product, multi-depot vehicle routing problem often present in a recyclable waste collection system. For medium and large scale problems, exact formulations are not capable to solve the problem since this is a hard combinatorial problem. This triggered off the development of a hybrid method that combines exact formulations with some heuristic procedures. The main idea of this method is to relax some constraints of the problem, and set the variables that do not violate the relaxed constraints. The exact formulations embedded in the hybrid algorithm are based on the two-commodity flow formulation for the Vehicle Routing Problem. The effectiveness of the proposed procedure is tested and shows that it is capable of producing the optimal solution in less time than the exact formulation in seven out of the eight test instances. Since the hybrid method proves to be effective, it is applied to a medium scale problem based on a real recyclable waste collection system. The detailed results for each module of the hybrid method are showed.

As further work, we intend to apply the proposed method to a larger problem, as well as incorporate a procedure to schedule the vehicle routes within the timeframe. Efforts to improve exact formulations performance will also be developed.

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