

Abstract Number: 015-0699

Abstract Title: A Multi-product, Multi-depot Periodic Vehicle Routing Problem in a Reverse Logistics System

Authors:

1) Tânia Rodrigues Pereira Ramos

ISCTE-IUL

Avenida das Forças Armadas, Edifício ISCTE, 1649-026 Lisboa, Portugal

CEG-IST, UTL

Avenida Professor Cavaco Silva, 2780-990 Porto Salvo, Portugal

tania.ramos@iscte.pt

+351 21 7903 415

+351 21 4233 533

2) Maria Isabel Gomes

CMA – FCT, Universidade Nova Lisboa

Campus da Caparica, 2829-516 Caparica, Portugal

mirg@fct.unl.pt

+351 21 2948388

3) Ana Paula Barbosa Póvoa

CEG-IST, UTL

Avenida Professor Cavaco Silva, 2780-990 Porto Salvo, Portugal

apovoa@ist.utl.pt

+351 21 4233 507

POMS 21st Annual Conference

Vancouver, Canada

May 7 to May 10, 2010

1 Introduction

Reverse logistics is becoming more important as recycling and environmental concerns gain significance. This new reality has a strong impact on supply chain design, since it no longer ends when the product is delivered to the final customer, but now also includes used products return.

Recycling involves processing used materials into new products, thus, a product life cycle does not end upon use or consumption. For products reaching the end of their working life, and changing from “products” to “waste”, a new cycle begins. The objective of this new cycle is to recover their remaining value by reinserting them into the supply chain (not necessarily the same one).

The recycling of packaging materials, imposed by the European Union (EU), has forced member states to develop new collection systems. The traditional routes defined for organic waste do not fit the particularities of packages: different vehicles, different collection rates, different bin locations.

This work will focus on the design of recyclable waste collection systems. Considering this waste stream, there are usually three types of materials used in packaging that can be recycled: glass, paper and plastic/metal. The final consumer is responsible to separate these materials and drop them into special containers. Those materials are then collected in a regular basis and taken to a treatment plant by the company responsible for the recyclable waste collection system. The design of such systems involves strategic, tactical and operational decisions. This work aims at supporting tactical decisions since it focus on the definition of service areas in collection systems with more than one depot. It is based on a

real case study: the recyclable waste collection system with five depots that covers the Alentejo region in southern Portugal.

In addition to the definition of the vehicle routes, it is also necessary to decide which containers are collected on each time unit (since the containers have different collection frequencies) and from which depot the collection is to be performed. This aspect adds two decision levels to the classical Vehicle Routing Problem, where more than one product is to be collected in different routes. The resulted problem is modelled as a multi-product, multi-depot periodic vehicle routing problem (MDPVRP). A mixed-integer linear programming model is developed and applied to some problem instances based on the real problem under study.

This paper is structured as follows. After a brief review of the literature on multi-depot periodic vehicle routing problem (MDPVRP) in Section 2, we described the real problem under study in Section 3. We characterise generically the model in Section 4 and describe the instances extracted from the real case and present the computational results in Section 5. Finally, we draw conclusions and discuss future work directions.

2 Literature Review

MDPVRP is usually defined by a graph $G=(V, A)$ over a planning horizon of t days. V is the vertex set and A is the edge set. The vertex V is partitioned into two subsets $V_d = \{v_1, \dots, v_n\}$ and $V_c = \{v_{n+1}, \dots, v_{n+p}\}$, representing respectively the set of depots and the set of cities or clients. At each depot are based k vehicles that have to visited p clients in a planning horizon of t days. Each client has specified a service frequency and a set of allowable combinations of visit days.

MDPVRP is a multilevel combinatorial optimization problem (Hadjiconstantinou and Baldacci, 1998). At the first level we need to define boundaries for each depot service area. At the second level, we need to decide which customers are visited on each day of the planning horizon. At the third level, we need to solve a classical VRP for each depot and for each day of the given period. Finally, at the fourth level, we need to solve a classical Travelling Salesman Problem for each route.

This problem consists of simultaneously selecting a set of visit days for each client, defining the service areas of each depot and establishing vehicle routes for each day of the planning horizon. Therefore, MDPVRP combines two problems: Multi-Depot Vehicle Routing Problem (MDVRP) and Periodic Vehicle Routing Problem (PVRP). These two problems have received a great deal of attention, but the combination of them has seldom been studied in the literature, and consequently only few models have been developed.

Hadjiconstantinou and Baldacci (1998) presented a heuristic approach based on tabu search for the MDPVRP. The heuristic algorithm is applied to a real case of a utility company that provides preventive maintenance services to a set of customers. This company had 17 vehicles, based on 9 depots, to serve 162 customers with a frequency that can vary from once a day to once every four weeks. The large scale problem motivated the authors to apply a heuristic algorithm instead of an exact algorithm. The problem was to determine the boundaries of the geographic areas served by each depot, the list of customers visited each day and the vehicle routes. This problem is multi-objective, since the company wanted to improve customer service (customer service is measured by the average frequency of visits to customers) and minimize total routing cost.

Parthanadee and Logendran (2006) presented a problem of delivering a set of products from several depots to a number of customers, under a scenario where product supplies are

limited, thus allowing for backordering. The authors provided the model formulation, the development of three tabu-search-based algorithms and proposed a fast technique to find a lower bound (the selective LP relaxation) to the multi-product, multi-depot PVRP. They also investigated the impact of allowing interdependent operations among depots when supplies are limited. The results obtained with the regular branch-and-bound method, the selective LP relaxation and with three tabu search heuristics were compared for small instances. The performance of the three tabu search algorithm is compared on small, medium and large problems.

For the MDVRP, as mentioned before, there are several models developed (exact and approximate approaches). Due to its NP-hard combinatorial factor, the models presented in the literature are mostly heuristics-based. There are still few exact algorithms in the literature. Laporte et al. (1984), as well as Laporte et al. (1988), have developed exact branch and bound algorithms for solving the symmetric and asymmetric version of the MDVRP, respectively. On the contrary, there are several heuristic algorithms developed to solve MDVRP (Tillman and Cain (1972), Golden, Magnanti and Nguyen (1977), Renaud et al.(1996), Salhi and Sari (1997), Lim and Wang (2005), Crevier et al. (2007), among others).

For the PVRP there are also several heuristic approaches. Beltrami and Bodin (1974), Russel and Igo (1979) and Teixeira et al. (2004) developed heuristic algorithms for the PVRP and applied to waste collection problems. Mourgaya and Vanderbeck (2007) presented a column generation procedure followed by a rounding heuristic to solve a PVRP with two objectives: minimizing total distance travelled and balance workload between vehicles. Other heuristic applications to PVRP can be found in Christofides and Beasley

(1984), Gaudioso and Paletta (1992), Chao et al. (1995), Cordeau et al. (1997) or Alonso et al. (2008), to name a few.

The literature review reveals that MDPVRP, MDVRP and PVRP are usually tackled by means of heuristics because of the hard combinatorial nature of this kind of problems. However, due to recent developments in integer programming software systems, the ability to solve hard combinatorial problems have been improved (see Atamturk and Savelsbergh (2005) for reviewing solvers state-of-the-art). For example, Fukasawa et al. (2006) and Baldacci et al. (2008) had recently developed exact methods to solve the classical Capacitated Vehicle Routing Problem (CVRP). The exact methods are applied over a set of instances with number of customers between 12 and 199 and number of vehicles up to 14.

3 Problem Description

A recyclable waste collection system is responsible to collect, within a certain geographic area, the recyclable materials dropped by the final consumer into special containers. Considering waste packaging, there are usually three types of materials used in packaging that can be recycled: glass, paper and plastic/metal. Therefore, the recyclable waste collection system provides three types of containers scattered over the geographic area. In terms of the number of containers placed in a collection site, three scenarios can be observed: one container of one recyclable material, one container for each of the three recyclable materials, or several containers of one or more recyclable materials. The collection sites have different collection frequencies depending on their location and on the recyclable materials to collect. For instance, the collection frequency of paper is higher than glass due to the product characteristics (a container with paper fills up more rapidly than a container with glass).

The number of depots and vehicles depends on the dimension of the geographic area that the system is responsible for. A big geographic area implies multiple depots, where vehicles start and end the collection routes; a small area may imply only one depot. The existence of multiple depots requires a service areas definition by depot: each depot is responsible to collect a set of collection sites and to define the collection routes. Usually, the vehicles used in the collection don't have compartments, so each recyclable material has to be collected in separated routes.

In Portugal, there are 35 recyclable waste collection systems (Sociedade Ponto Verde, 2010). This work focused on the company responsible for the recyclable waste collection network covering 7 municipalities of the Alentejo region, southern Portugal (Figure 1). This company owns and operates 5 depots and 1612 recyclable waste containers (651 glass bins, 513 paper bins, and 448 plastic/metal bins), that are clustered in 212 collection sites (a collection site, as mentioned before, aggregates one or more containers of one or more recyclable materials – for example, the city “Alcácer do Sal” is one collection site, where are 37 glass bins, 29 paper bins and 21 plastic/metal bins). The collection is performed by a heterogeneous fleet of 7 vehicles (driven by seven different drivers). Every depot has one vehicle, except the depot located in Santiago do Cacém that has two vehicles. The planning horizon has four weeks since the lowest collection frequency found was once monthly. The company works 5 days per week, 8 hours per day. See Figure 1 with the summary of the real problem characteristics.

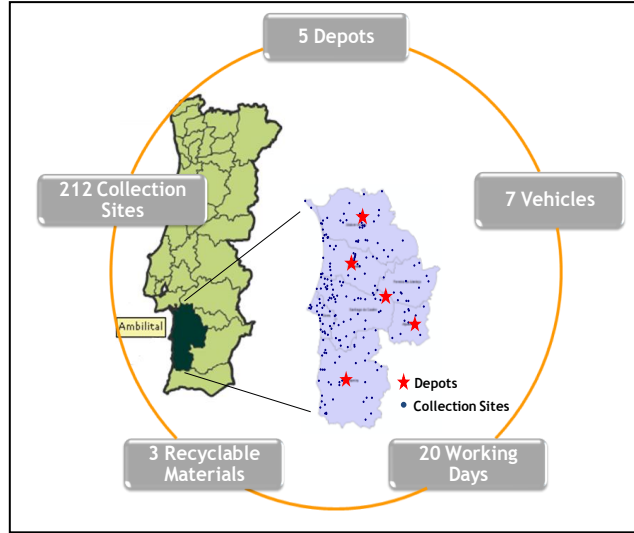


Figure 1: Real problem description

4 Model approach

A model formulation for the multi-product, multi-depot periodic recyclable waste collection routing problem was developed. The decision variables are x_{ijmkt} and $y_{i,m,pad}$, which represent the routing solution and the collection pattern assignment, respectively. $x_{ijmkt} = 1$ if site j is visited immediately after site i , to collect material m , by vehicle k , on its u^{th} trip in day t ; 0 otherwise. $y_{i,m,pad} = 1$ if collect pattern pad is assigned to collection site i with material m ; 0 otherwise. The objective function of the model focuses on minimizing the total distance travelled to collect all recyclable materials at collection sites over the timeframe:

$$\text{Min} \sum_{i \in I} \sum_{j \in I} \sum_{m \in M} \sum_{k \in K_n} \sum_{u \in U} \sum_{t \in T} d_{ij} x_{ijmkt} \quad (1)$$

where d_{ij} is the distance between node i and node j .

The model takes into account the classical routing restrictions which impose that (i) a vehicle cannot leave and return to a depot other than its home depot, (ii) the vehicle capacity and the duration of each trip are not exceed and (iii) the route continuity. This model has also assignment constraints which imposes that (iv) one feasible collect pattern must be assigned to each collection site with material m and (v) each collection site is

visited only on the days corresponding to the collect pattern assigned. To prevent subtours we added a transit load constraint (Ropke et al. 2007), which is based on the Miller-Tucker-Zemlin (1960) formulation to eliminate subtours.

For this specific problem, there is the need to add three new constraints: (vi) one assures that at each collection site, all recyclable materials are collected from the same depot; (vii) one that verifies if the duration of all trips of each vehicle on each day doesn't exceed the imposed limit and (viii) one that ensures the u^{th} collect trip cannot be initialized unless the previous trip, the $(u-1)^{\text{th}}$ trip, has been constructed.

5 Test Results

To test the model developed we generate five instances based on the real case under study. These instances reflect parts of the real problem. The structure of these instances is presented in Table 1.

Table 1: Structure of the test instances

<i>Instance</i>	<i>N° of Depots</i>	<i>N° of Collection Sites</i>	<i>N° of recyclable materials</i>	<i>N° of vehicles</i>	<i>N° of trips</i>	<i>N° of days</i>
1	2	8	3	2	2	10
2	2	13	3	2	2	10
3	3	17	3	3	2	10
4	3	27	3	3	2	10
5	3	37	3	3	3	10

The location of the collection sites and depots in the five instances are represented in Figure 2.

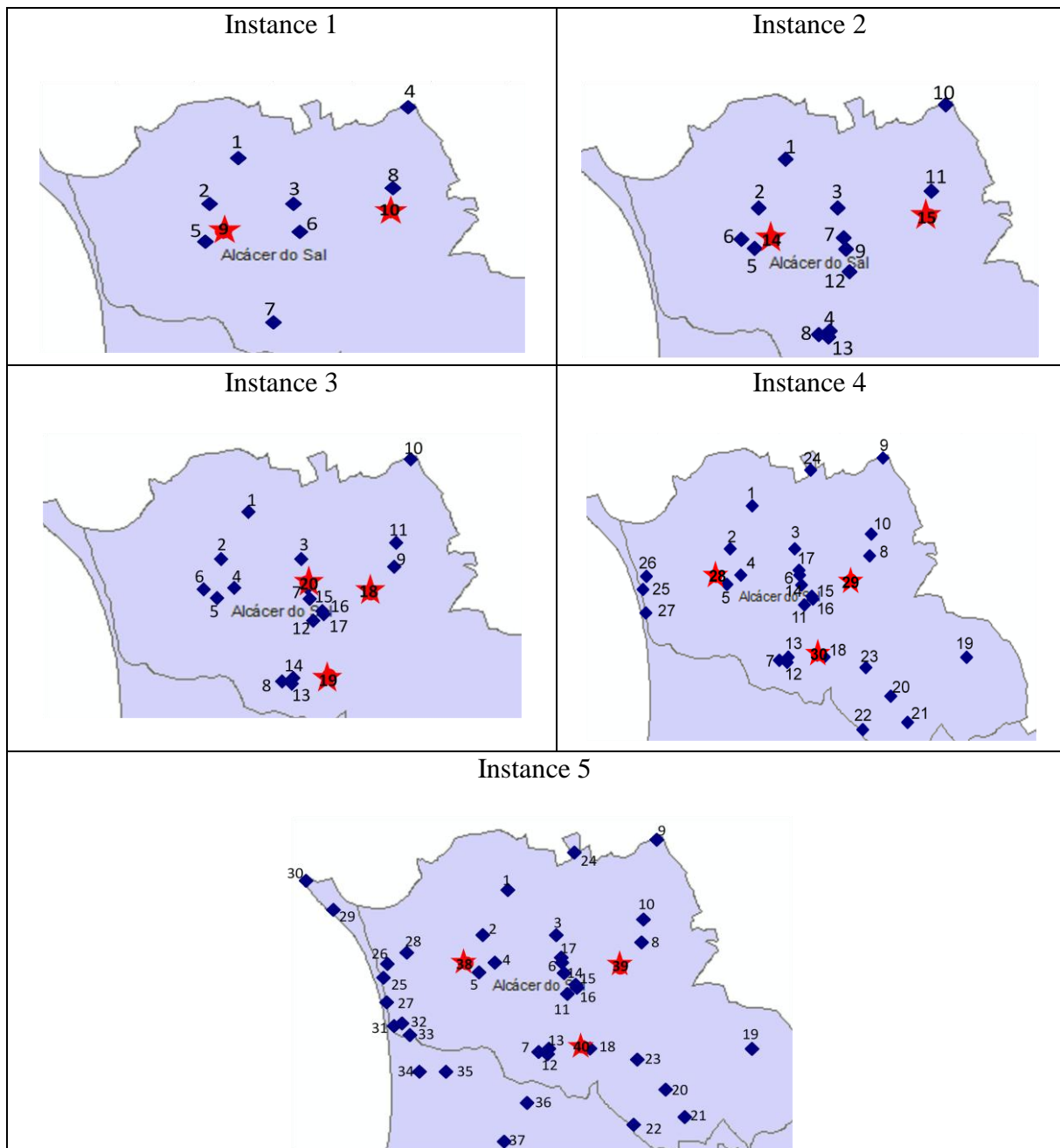


Figure 2: Location of the collection sites and depots in the five instances

The instances are solved using the branch-and-bound method implemented in the solver of the CPLEX Optimizer 12.1.0. A Intel(R) Core(TM) i7 CPU 920 @ 2,66 GHz is used.

The total number of constraints and variables of the test instances are presented in Table 2.

Table 2: Number of variables and constraints of the test instances

<i>Instance</i>	<i>Number of constraints</i>	<i>Number of variables</i>	
		<i>Discrete</i>	<i>Total</i>
1	12 518	8 470	13 271
2	27 695	19 510	28 871
3	69 452	47 611	75 692
4	158 047	103 206	167 647
5	423 060	228 760	443 141

Due to the combinatorial nature of the problem in analysis, the number of constraints and variables grows exponentially with the increasing number of nodes. This fact has a negative impact in the computational time taken to find the optimal solution of large instances. Therefore the problem is decomposed with respect to recyclable materials. In order to access the quality of the obtained solutions, these are compared with the solutions obtained by solving the model globally.

Since the three recyclable materials are to be collected in separate routes, and all recyclable materials at each collection site must be collected by a vehicle from the same depot, the method of decomposition is based on recyclable materials. Therefore, the model is run as follows: the first iteration has only the data concerning one of the recyclable material; the results of this iteration are feed to the second iteration in the form of parameters, the variables concerning the second material are now optimized assuming the solution of the previous iteration; finally, the solution regarding the third material is optimized keeping unchanged the previously computed values.

To choose the recyclable materials sequence to enter the model we have done some tests and observed that the first material in the sequence should be the paper and the order of the other two materials was less relevant. The material paper is the more restrictive one because it has the higher collection frequency. Considering the entire planning horizon, there will be more routes to collect paper than to collect the two other materials. One of the

consequences of this decomposition method is that services areas are defined in the first iteration (with paper routes) and remain unchanged over the remaining iterations.

The results that follow assume two different sequences. The first is Paper → Glass → Plastic/Metal (see Figure 3). The second sequence is Paper → Glass and Plastic/Metal (see Figure 4). With this last sequence only two iterations are need to solve the problem.

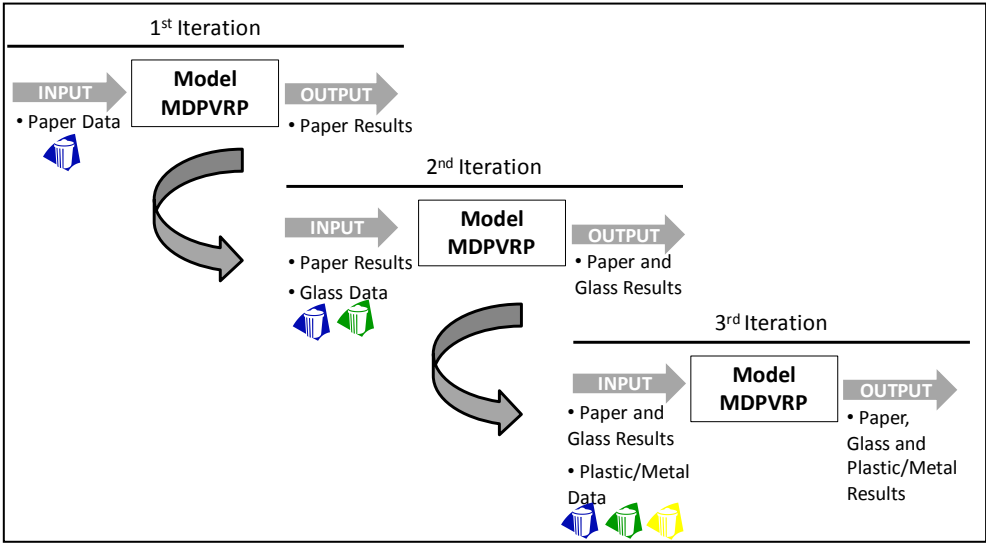


Figure 3: Model decomposition by recyclable material (3 iterations)

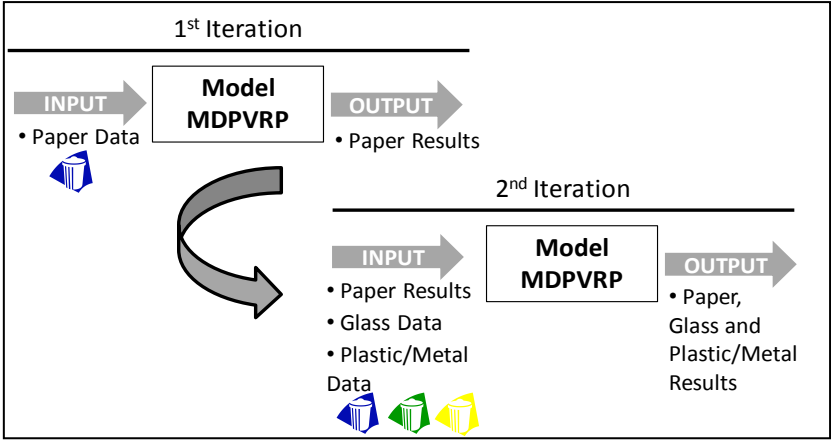


Figure 4: Model decomposition by recyclable material (2 iterations)

The results for the five instances are presented in Table 3, divided by the method used: Global (all products at once); Decomposing by Material (3 iterations) and Decomposing by

Material (2 iterations). In all runs, the computational time is arbitrarily limited to 7 hours (25 200 secs).

Table 3: Results obtained for the five instances

	Opt. Value (Km's)	CPU (seconds)	GAP (%)
Instance 1 Global	517	104	0
Decomposing by Material (3 iterations)			
1) Paper	296	0,7	0
2) Paper and Glass	375	0,9	0
3) Paper and Glass and Plastic/Metal	517	0,9	0
Decomposing by Material (2 iterations)			
1) Paper	296	0,7	0
2) Paper, Glass and Plastic/Metal	517	5	0
Instance 2 Global	1001	25200	33,2
Decomposing by Material (3 iterations)			
1) Paper	540	210	0
2) Paper and Glass	652	202	0
3) Paper and Glass and Plastic/Metal	1032	7	0
Decomposing by Material (2 iterations)			
1) Paper	540	210	0
2) Paper, Glass and Plastic/Metal	1020	25200	5,8
Instance 3 Global	990	25200	29,2
Decomposing by Material (3 iterations)			
1) Paper	560	14	0
2) Paper and Glass	704	25	0
3) Paper and Glass and Plastic/Metal	990	764	0
Decomposing by Material (2 iterations)			
1) Paper	560	14	0
2) Paper, Glass and Plastic/Metal	990	25200	1,8
Instance 4 Global	2057	25200	50,7
Decomposing by Material (3 iterations)			
1) Paper	780	25200	4,5
2) Paper and Glass	996	25200	3,4
3) Paper and Glass and Plastic/Metal	1430	25200	4,5
Decomposing by Material (2 iterations)			
1) Paper	780	25200	4,5
2) Paper, Glass and Plastic/Metal	1430	25200	9,8
Instance 5 Global	-	-	-
Decomposing by Material (3 iterations)			
1) Paper	1036	25200	21
2) Paper and Glass	1330	25200	5,2
3) Paper and Glass and Plastic/Metal	2032	25200	11,4
Decomposing by Material (2 iterations)			
1) Paper	1036	25200	21
2) Paper, Glass and Plastic/Metal	2086	25200	18,5

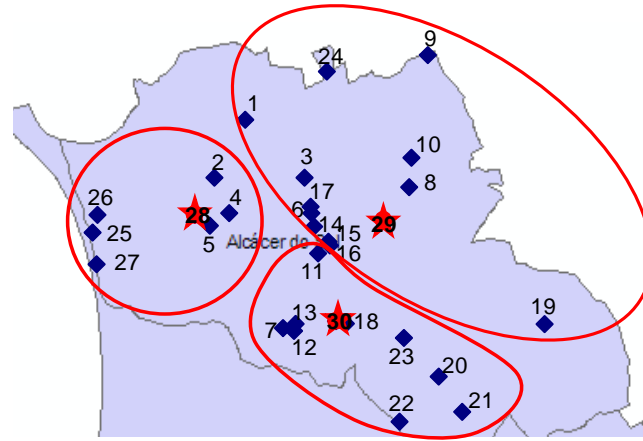
Opt. Value is the objective function value; the “-“ implies that no integer solution was found by CPLEX within the 7 hours limit.

Instance 1 was the only one where the three methods gave the same solution with the same gap of 0%. However, the three methods have significant differences in computation time. In this instance, decomposing by material with 3 iterations reveals to be very efficient and

effective (this method finds and proves the optimal solution in less time than the other two methods). On the contrary, in instance 2, the methods of decomposition were not capable to find the optimal solution. The service areas defined by paper routes on the first iteration lead to a worse final solution than when we solve the model with all materials at once. Despite the solution found by solving the model globally is better than the solutions found by decomposition, this method was not able to prove the optimal solution within the 7-hour time limit (gap of 33%). In instance 3, the best solution was obtained by decomposing by material with 3 iterations (the same optimal value with less time than the other two methods). For instance 4, despite not proving the optimal solution, low gaps are achieved with the decomposition method with 3 iterations. The solution found by solving the model globally was worse than applying the decomposition methods. For instance 5, both methods of decomposition find solutions, although with 3 iterations is found a better solution, with a lower gap than with 2 iterations. However, these solutions present high gaps. Solving the model globally failed to prove the optimal solution for instance 2, 3 and 4, and failed to report an integer solution within the 7 hour limit for instance 5.

In order to illustrate the results provided by the model, Figure 4 shows with detailed the solution for instance 4. The three service areas corresponding to each depot are marked in the map, which origin a total distance travelled to collect the three recyclables materials over the 10 days of 1430 km's. The collection frequency for recyclable material paper is 4, so the collection routes are repeated 4 times in the planning horizon; for glass is 1 and for plastic/metal is 2. The routes are scheduled in a 10 days planning horizon according to the collections patterns allowed. Each vehicle could do upon to 2 trips in a day. A collection site, as mentioned before, may not have all the three materials; for example, collection site "11" has only containers of plastic/metal, so it only appears on plastic/metal routes.

Service Areas



Total Distance Travelled: 1 430 Km's

Routes by Material and by Day

Depot 28

Days

Materials	1	2	3	4	5	6	7	8	9	10
Paper	28→26→25 →27→28 28→5→4→ 2→28			28→26→25 →27→28 28→5→4→ 2→28			28→26→25 →27→28 28→5→4→ 2→28			28→26→25 →27→28 28→5→4→ 2→28
Glass		28→5→4→ 2→28						28→26→25 →28		
Plastic/ Metal		28→27→25 →26→28 28→4→2→ 28				28→27→25 →26→28 28→4→2→ 28				

Depot 29

Days

Materials	1	2	3	4	5	6	7	8	9	10
Paper	29→8→10 →19→29 29→15→16 →14→17→ 3→1→24→ 29			29→8→10 →19→29 29→15→16 →14→17→ 3→1→24→ 29			29→8→10 →19→29 29→15→16 →14→17→ 3→1→24→ 29			29→8→10 →19→29 29→15→16 →14→17→ 3→1→24→ 29
Glass						29→3→1→ 24→9→29 29→19→10 →17→6→ 14→16→15 →29				
Plastic/ Metal		29→8→9→ 10→19→29			29→3→1→ 17→6→14 →15→29	29→19→10 →9→8→29			29→3→1→ 17→6→14 →15→29	

Depot 30

Days

Materials	1	2	3	4	5	6	7	8	9	10
Paper	30→18→30 30→13→7 →12→22→ 21→20→23 →30			30→18→30 30→13→7 →12→22→ 21→20→23 →30			30→18→30 30→13→7 →12→22→ 21→20→23 →30			30→18→30 30→13→7 →12→22→ 21→20→23 →30
Glass									30→13→7 →12→22→ 21→20→23 →18→30	
Plastic/ Metal		30→11→7 →13→30 30→22→21 →20→23→ 30	30→18→30			30→13→7 →11→30 30→22→21 →20→23→ 30		30→18→30		

Figure 4: Detailed solution for instance 4

6 Conclusions

In this work we presented a reverse logistics real problem that, due to its characteristics, was model as a multi-product, multi-depot periodic vehicle routing problem. This is an NP-hard combinatorial problem, which have been tackled by heuristic algorithms. Due to the recent improvements on commercial optimizers, it is now possible to solve hard combinatorial problems by exact algorithms.

Five small and medium size scale instances, based on the real problem, were generated to test the model developed. Besides solving the model globally, we want to test a decomposition method by material, which solves the model in two or three iterations.

Solving the model decomposing by material with three iterations reveals to be a very efficient method. This method finds good solutions (in some instances, the optimum solution) in a very reasonable computation time considering the tactical nature of the decisions. Decomposition by material with two iterations is more time consuming, and only lead to a better solution in instance 2. For remain instances, this method find the same or a worse solution.

When the problem is solved globally, good solutions are found for instances 1, 2 and 3 but are not proven as optimal within the 7 hours limit (except for instance 1). In the larger instances, this method was not capable to find a good solution (instance 4) or an integer solution (instance 5).

As future work, other formulations and other methods of decomposition will be tested in order to decrease the obtained gaps for the medium scale instances and to solve the real case presented.

7 References

- Alonso, F., M. J. Alvarez and J. E. Beasley (2008). "A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions." Journal of the Operational Research Society 59(7): 963-976.
- Atamturk, A. and M. W. P. Savelsbergh (2005). "Integer-programming software systems." Annals of Operations Research 140(1): 67-124.
- Baldacci, R., N. Christofides and A. Mingozzi (2008). "An exact algorithm for the vehicle routing problem based on the set partitioning formulation with additional cuts." Mathematical Programming 115(2): 351-385.
- Beltrami, E.J. and L.D. Bodin (1977). "Networks and Vehicle Routing for Municipal Waste Collection." Networks 4: 65-94.
- Chao, I. M., B. L. Golden and E. Wasil (1995). "An Improved Heuristic for the Periodic Vehicle-Routing Problem." Networks 26(1): 25-44.
- Christofides, N. and J. E. Beasley (1984). "The Period Routing Problem." Networks 14(2): 237-256.
- Cordeau, J. F., M. Gendreau and G. Laporte (1997). "A tabu search heuristic for periodic and multi-depot vehicle routing problems." Networks 30(2): 105-119.
- Crevier, B., J. F. Cordeau and G. Laporte (2007). "The multi-depot vehicle routing problem with inter-depot routes." European Journal of Operational Research 176(2): 756-773.
- Desrochers, M. and G. Laporte (1991). "Improvements and Extensions to the Miller-Tucker-Zemlin Subtour Elimination Constraints." Operations Research Letters 10(1): 27-36.
- Fukasawa, R., H. Longo, J. Lygaard, M. P. de Aragao, M. Reis, E. Uchoa and R. F. Werneck (2006). "Robust branch-and-cut-and-price for the capacitated vehicle routing problem." Mathematical Programming 106(3): 491-511.
- Gaudioso, M. and G. Paletta (1992). "A Heuristic for the Periodic Vehicle-Routing Problem." Transportation Science 26(2): 86-92.
- Golden, B. L., T. L. Magnanti and H. Q. Nguyen (1977). "Implementing Vehicle Routing Algorithms." Networks 7(2): 113-148.
- Hadjiconstantinou, E. and R. Baldacci (1998). "A multi-depot period vehicle routing problem arising in the utilities sector." Journal of the Operational Research Society 49(12): 1239-1248.
- Laporte, G., Y. Nobert and S. Taillefer (1988). "Solving a Family of Multi-Depot Vehicle-Routing and Location-Routing Problems." Transportation Science 22(3): 161-172.

- Lim, A. and F. Wang (2005). "Multi-depot vehicle routing problem: A one-stage approach." Ieee Transactions on Automation Science and Engineering 2(4): 397-402.
- Miller, C. E., A. W. Tucker and R. A. Zemlin (1960). "Integer Programming Formulation of Traveling Salesman Problems." Journal of the Acm 7(4): 326-329.
- Mourgaya, M. and F. Vanderbeck (2007). "Column generation based heuristic for tactical planning in multi-period vehicle routing." European Journal of Operational Research 183(3): 1028-1041.
- Parthanadee, P. and R. Logendran (2006). "Periodic product distribution from multi-depots under limited supplies." Iie Transactions 38(11): 1009-1026.
- Renaud, J., G. Laporte and F. F. Boctor (1996). "A tabu search heuristic for the multi-depot vehicle routing problem." Computers & Operations Research 23(3): 229-235.
- Ropke, S., J. F. Cordeau and G. Laporte (2007). "Models and branch-and-cut algorithms for pickup and delivery problems with time windows." Networks 49(4): 258-272.
- Russel, R. and W. IGO (1979). "An assignment routing problem." Networks 9: 1-17.
- Salhi, S. and M. Sari (1997). "A multi-level composite heuristic for the multi-depot vehicle fleet mix problem." European Journal of Operational Research 103(1): 95-112.
- Sociedade Ponto Verde (2010). url: www.pontoverde.pt, access on 18/02/2010
- Teixeira, J., A. P. Antunes and J. P. de Sousa (2004). "Recyclable waste collection planning - a case study." European Journal of Operational Research 158(3): 543-554.
- Tillman, F. A. and T. M. Cain (1972). "Upperbound Algorithm for Single and Multiple Terminal Delivery Problem." Management Science Series a-Theory 18(11): 664-682.