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Interdisciplinary tasks: Pre-service teachers' choices and approaches

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Abstract: This study focusses on the criteria used by pre-service teachers of Mathematics to choose interdisciplinary tasks. The pre-service teachers' knowledge is assumed as the basis of the actions taken and used as the origin of the choices and approaches observed. The study adopts a qualitative and interpretative methodology and the data were collected using class observation and interviews. The analysis is guided by the *Application and Pedagogical Content Knowledge*, a model inspired on TPACK (from Mishra and Koehler) and MKT (from Ball and colleagues). The conclusions point to an appreciation of the mathematical part of the tasks and to a devaluation of the remaining components. This suggests difficulty in articulating and integrating different domains of knowledge and points to a fragmented view of the potential of using mathematical applications.

Keywords: applications, mathematics, pre-service teachers, teachers' knowledge.

Introduction

Most of the problems we can find in the reality are related to more than one area of disciplinary knowledge. This means they can be considered as interdisciplinary problems. However, when these problems are approached in school, that approach tends to focus on the knowledge of one specific subject. Many times, the procedural aspects of that specific subject become the central part of the problem and everything else is ignored, simplified or even removed from what was a real situation. The result can be a task that no longer is a problem, neither an interdisciplinary task.

In this study, a set of tasks with a real context was given to a class of pre-service teachers taking a course on didactics of mathematics at a master program for future secondary mathematics teachers. The pre-service teachers were asked to choose three tasks from that set of tasks and develop a lesson plan explaining how they will use the tasks in lessons of mathematics.

The main goal of the study is to characterize the pre-service teachers' choice of tasks and the related professional knowledge. It was specifically intended to: (1) identify the criteria used by the pre-service teachers to select the tasks and (2) analyze what these options suggest in terms of their professional knowledge. The pre-service teachers' knowledge is assumed as the basis of the actions taken and an analysis of the tasks chosen and the related reasons is assumed as a way to access their professional knowledge.

Theoretical framework

Interdisciplinarity

The literature offers a variety of ways of understanding interdisciplinarity. Drake (1991) considers the integration of two or more disciplines, assuming interdisciplinarity as one type of integration. And Williams et al. (2016) describe this type of integration as an approach where two or more disciplinary contents are considered at the same time. This means that we are considering mathematics and some other(s) content(s) simultaneously, while addressing a specific topic or theme, but in such a way that all the disciplines keep their specific nature. Other types of integration are present, for instance, in a multidisciplinary or transdisciplinary approach (Williams et al., 2016).

Roth (2014, p. 317), states that: "interdisciplinarity denotes the fact, quality, or condition of two or more academic fields or branches of learning". So, in these case, the meaning ascribed to interdisciplinarity is similar to the one ascribed to integration by Drake, referring in a broad way to a situation where some kind of integration between disciplines occurs, regardless of the characteristics of that integration. This means that depending on the author, integration can be a synonymous of interdisciplinarity or interdisciplinarity can be a special kind or level of integration.

The diversity of ways how two disciplines can be articulated was the focus of attention of several authors, who devoted their work to analyse its characteristics and develop classifications. That is the case of Fogarty (1991), who considered ten levels, according to the degree of articulation between the disciplines: fragmented, connected, nested, sequenced, shared, webbed, threaded, integrated, immersed, and networked.

Mathematical modelling or application is another term usually associated with interdisciplinarity (these two terms are often considered together and here we are not going to distinguish between them). However, Ferri and Mousoulides (2017) alert us to the need to carefully reflect on the understanding of these two concepts - modelling/application and interdisciplinarity -, in order to identify what is common to them and what is different. The authors consider that the usual definitions of these terms show strong overlaps, not always making clear the differences. Mathematical modelling or application is often presented as an activity involving articulation between reality and mathematics (Ferri & Mousoulides, 2017). And this means that modelling/application always requires a real-life context, but real-life situations tend to be related to some other discipline besides mathematics. In this sense, modelling/application is always an interdisciplinary activity. According to Ferri and Mousoulides (2017), modelling/applications are activities set within authentic contexts, and offering opportunities to engage in important mathematical processes, such as describing, analysing, constructing, and reasoning.

Nowadays, STEM has become the ultimate form of interdisciplinarity (Williams et al., 2016). However, STEM and interdisciplinarity are not the same thing. Bergsten and Frejd (2019) present STEM as an interdisciplinary approach to learning, based on real world situations from Science, Technology, Engineering, and Mathematics. This means STEM presupposes interactions within a specific set of disciplines, leaving out all the others, such as the ones related to, for instance, Humanities or Economics. Bergsten and Frejd (2019) defined STEM as based on real world situations, and Ferri and Mousoulides (2017) characterize mathematical modelling/application as an activity involving articulation between reality and mathematics. However, once again, not all the real situations are related to Science, Technology, and Engineering, what means that STEM is also not the same as modelling. Therefore, STEM is an interdisciplinary approach, often requiring some modelling process.

Here we will assume a mathematical application as a task requiring some articulation between reality and mathematics, which will include some kind of interdisciplinary approach.

Teachers' knowledge

Several authors have developed models of teachers' knowledge, identifying and characterizing different domains that integrate that knowledge (Ruthven, 2011). As a consequence, there are several characterizations of teacher professional knowledge developed over the years but, whether we base ourselves on the work of Shulman (1986) or on some other more recent work, all characterizations emphasize the importance of different types of teachers' knowledge.

Hill and Ball (2009) draw on Shulman's work and conceptualized Mathematical Knowledge for Teaching (MKT), where they consider two major areas: SMK - Subject Matter Knowledge and PCK - Pedagogical Content Knowledge (see Figure 1). In the scope of the first, they consider CCK - Common Content Knowledge, SCK - Specialized Content Knowledge and KMH - Knowledge at the Mathematical Horizon. And in the context of the second, they consider KCS - Knowledge of Content and Students, KCT - Knowledge of Content and Teaching and KC - Knowledge of Curriculum.

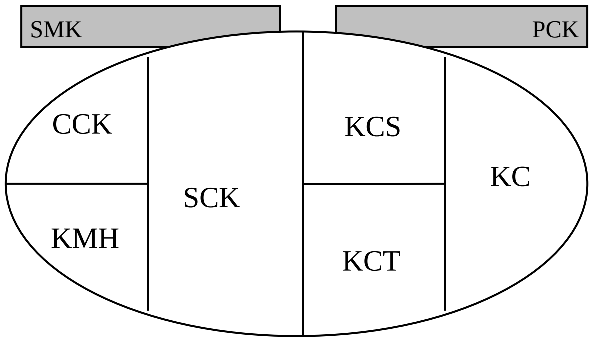


Figure 1. MKT – Mathematical Knowledge for Teaching (Hill & Ball, 2009)

In SMK, Ball, Thames and Phelps (2008) consider Common Content Knowledge (CCK), which is identical to the knowledge used in other professions where mathematical knowledge is required. It includes knowledge of the definition of a concept or object, how to perform a procedure and how to solve mathematical tasks correctly and even identify the correctness of an answer given by others. The authors also consider a teachers' specific knowledge, the Specialized Content Knowledge (SCK). This is a core knowledge in this model and corresponds to a knowledge that is used in the classroom and that is necessary for the teacher to be able to teach efficiently. It corresponds, for example, to the knowledge needed to identify the source of some error. A third type of knowledge was introduced later by Hill and Ball (2009), the Knowledge at the Mathematical Horizon (KMH). It corresponds to what they describe as a kind of peripheral vision needed to teach. This knowledge encompasses a broad and comprehensive overview of the mathematics teaching landscape, including an awareness of the mathematical topics covered in previous years and how they relate to those approached at the present and in the future.

In addition to these domains of knowledge, Ball, Thames and Phelps (2008) and Hill and Ball (2009) also refer to a certain integration of content knowledge with another type of knowledge. They thus refer to three domains, that we approach here in a very brief way: Knowledge of Content and Student, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum.

Knowledge of Content and Students (KCS) combines knowledge of students and mathematics. This knowledge corresponds to the ability to anticipate students' difficulties, respond to their thoughts and respond to them in a timely and convenient manner. It also includes making an appropriate choice of examples and representations to teach. Both in the preparation process and in the course of its implementation, the teacher must be aware of the conceptions held by the students regarding the topic under study and, in particular, regarding the students' misconceptions.

Knowledge of Content and Teaching (KCT) articulates knowledge about mathematics and teaching. It refers, among other elements, to the teacher's decisions regarding the sequence of activities, his awareness of the possible advantages and disadvantages of the representations used for teaching, his decisions about when to interrupt a discussion in class to clarify some aspects or to use a student's opinion, and also about how to call attention to a certain mathematical aspect.

Teachers' knowledge also includes knowledge of the curriculum, as well as how the different contents interrelate and evolve throughout the school year syllabus.

One of the main points of this model is the way how it emphasizes the mutual influence among different domains of the knowledge. That is, for instance, how the teachers' knowledge of the students impacts the mathematical content and how the teaching approach also impacts the mathematical content.

The integration of something new on the teachers' practice has proved to be challenging and to require some change on the professional knowledge. One of the most studied situations is the integration of technology. In this case, the need for a different knowledge is deeply recognized and the starting point for the development of some models intending to characterize the teachers' knowledge required to integrate it. One of the most well-known models in these circumstances is the TPACK from Mishra and Koehler (2006).

Mishra and Koehler (2006) argue that the articulation of technology knowledge with other types of knowledge is fundamental. According to the authors, the relationships between content, pedagogy and technology are complex and take multiple forms. Indeed, on the one hand, technology has its own imperatives that affect the content to be addressed and its representations and, on the other, it interferes with instructional options and other pedagogical decisions. Decision making regarding the use of technology thus has implications for other areas and as such it does not seem appropriate to consider it in isolation from pedagogical knowledge and content knowledge. Mishra and Koehler (2006) then propose a model that not only considers the three referred domains of knowledge (basic knowledge), but also addresses the connections, interactions and constraints that are established between them. Thus, they consider a Technological Pedagogical and Content Knowledge (TPACK), which is based on content knowledge, pedagogical knowledge and technological knowledge, but also respond to the influences of each of the basic knowledge on each other. They thus refer to Technological Content Knowledge (TCK), Technological Pedagogical Knowledge (TPK) and Pedagogical Content Knowledge (PCK). These three areas of knowledge are the essence of this model and what truly distinguishes it from others previously proposed.

Content Knowledge (CK), Pedagogical Knowledge (PK) and Pedagogical Content Knowledge (PCK) are, taking into account the origin of the model, consistent with the respective notions presented by Shulman and well documented in the literature.

Technology Knowledge (TK) involves the capabilities required to operate a technology and essentially consists of knowing how it works.

Technological Content Knowledge (TCK) is directly linked to how technology and content influence each other. It is a knowledge that, while relying on content knowledge, is different from this. Access to technology not only allows access to different representations, but also facilitates the connection and transition between them. As so, the teacher needs to know not only the content to be taught but also how it can be modified by the use of technology.

Technological Pedagogical Knowledge (TPK) is a knowledge related to the potentialities of technology and the way how teaching can be changed according to the use of it. It includes understanding how a given technology can enhance the accomplishment of a certain type of task, becoming familiar with a set of strategies that allow the students to exploit technology's capabilities, and knowing how to tailor certain teaching methods to integrate technology.

Technological Pedagogical and Content Knowledge (TPACK) is a knowledge developing from the three base components of the model (content knowledge, pedagogy and technology), but goes beyond these. This knowledge is different from that held by a mathematician or a technology expert and equally distinct from the general pedagogical knowledge shared by teachers of different subjects. It is the basis of effective technology integration and requires an understanding of concepts within technology and an understanding of pedagogical techniques that use technology constructively to teach concepts. It also requires a knowledge of what makes a concept difficult or accessible, and how technology can be used to promote students' learning. It also requires a sense of students' prior knowledge and how they learn, as well as a knowledge of how technology can be used to develop existing knowledge or to achieve new knowledge.

Quality teaching thus requires the development of an understanding of the complex relationships between the three base knowledge of the model and the ability to use that understanding to develop an appropriate and context-specific set of strategies.

At TPACK model, besides considering the relevance of the mathematical knowledge, of the pedagogical knowledge and of the technology knowledge, the mutual influence among these domains of knowledge is central. This means that when the technology becomes available, the teacher needs to consider the way it can impact the mathematics and the pedagogical approach. In these circumstances, it seems reasonable to admit that introducing mathematics applications requires some similar development of the teachers' professional knowledge. As so, we propose the model APCK - Application and Pedagogical Content Knowledge (see Figure 2), a conceptualization of the teachers' knowledge similar to the one developed at TPACK.

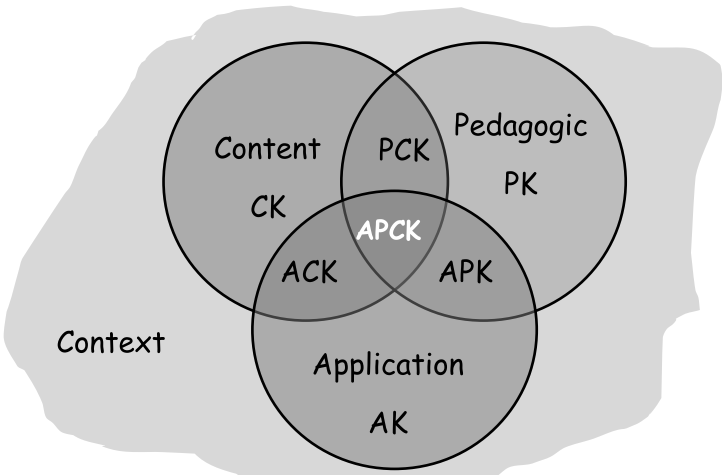


Figure 2. Application and Pedagogical Content Knowledge – APCK

On this model, Application and Content Knowledge (ACK) is directly linked to how the use of Mathematical Applications and the content influence each other. The use of mathematical applications promotes a different use of the mathematical knowledge, where the topics no longer are approached in a specific organized way. When working on mathematical applications all the mathematical knowledge of the students can be used at any time. As a result, the mathematics content no longer is addresses in a compartmented way. On the contrary, all the students' mathematics knowledge can be useful all the time. The consequence is an impact from the use of applications on the mathematical content. In the same way, the Application Pedagogical Knowledge (APK), is directly linked to how the use of Mathematical Applications and the Pedagogical approach influence each other. The use of mathematical applications requires a different approach, where options such as collaborative work, discussion of different approaches and presentation of the work developed are central.

Mathematical applications imply some real context and some interdisciplinary related knowledge. As so, Application Knowledge includes knowledge of disciplines beyond Mathematics, and this knowledge is not necessarily mastered by the teacher. The need to look for additional knowledge on fields outside mathematics requires a different attitude, appealing namely to reflection and critical reasoning. Reflection and critical reason are important skills for all mathematics' students and can be developed without using mathematical applications, the point is that applications turn them in central skills. And, once again, applications impact how the students learn mathematics (APK) and how the students come to think about what mathematics is (ACK).

Methodology

The study adopted a qualitative and interpretative methodology (Bryman, 2004). The data for the two case studies were collected by observation (two lessons) of a course of the master program for pre-service secondary school mathematics teachers and by an interview (after lessons). The analysis is based on the APCK model and intends to identify the domains of knowledge emphasized on each of the options assumed (based on the actions taken during the lessons and on the justifications presented for them on the interviews). As a consequence of the framework, a special attention is given to the mobilization of isolated or interlinked domains of knowledge. The analysis started from the identification of the reasons presented by the pre-service teachers for the options assumed and proceed relating these reasons to knowledge domains on APCK model.

A set of seven tasks, suitable for 10^{th} grade students, was given to the pre-service teachers: (1) *The construction of the gutter*, (2) *Folding the corner of a sheet*, (3) *Slalom*, (4) *The trains*, (5) *Apples*, (6) *The colony of bacteria*, (7) *The box*. They were then asked to choose three of these tasks and to develop lessons plans based on those tasks. Their decisions should be guided by the potential they ascribe to each of the tasks to promote the students' exploration of a situation from reality. Afterwards, the pre-service teachers comment on the reasons that guided their choices, justifying why they choose (or not) each of the tasks. Being the criteria for choosing or excluding a task based on the way how it relates to reality, the tasks were chosen to include different circumstances, as justified on the next section.

Tasks and pre-service teachers' choices and approaches

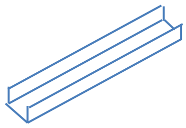
This section includes a brief description of the tasks and of the options of the group AB. This group includes two pre-service teachers, here designated by pre-service teacher A and pre-service teacher B, chosen because of their determination in their choices.

The three tasks chosen by this group from the set of seven tasks was (2) *Folding the corner of a sheet*, (4) *The trains*, and (5) *Apples*.

The construction of the gutter was the 1^{st} task (see Figure 3), and it was included on the set of tasks because it was a very open task, clearly including a context from reality, where the students need to analyze the situation and take decisions about what to do. There is a need to think about the function of a gutter, calling for some interdisciplinarity between mathematics and engineering, or some application of mathematics on the field of engineering.

(1) The construction of the gutter

One company produces rectangular metal plates of 30 cm wide. Joining these plates, the company constructs gutters to be placed on the roofs of the houses. The plates are constructed by folding equal sidebands, which form right angles with the base, as shown in the following figure. Many different folds can be made.



Your job is to study the situation and propose to the company the best measure for the folding of the plates, presenting the reasons that led to your decision.

Suggestion: you can begin by analyzing the capacity of the gutter in function of the folding that is made.

(adapted from Neves, Guerreiro, Leite, & Silva, 2010, p. 62)

Figure 3. Task 1: The construction of the gutter

This group did not choose task (1) *The construction of the gutter* because it was assumed as a not very well-defined task. According to the pre-service teachers: "We think this task is very vague about what the students are meant to do. So, we think they would have many difficulties and therefore we did not choose it".

The pre-service teachers think that the task is not clear enough about what the students are supposed to do and, as a consequence, they are afraid that the students can move away from what is intended and lose the focus on the mathematical content that they are supposed to work.

Folding the corner of a sheet was the 2nd task (see Figure 4), and it was included in the set of tasks because it requires data collection (students are asked to folder the sheet of paper and register some measures), however it cannot be considered exactly as a situation from reality or an application of mathematics.

(2) Folding the corner of a sheet

Fold a sheet of paper so that the upper left corner touches the underside of the sheet as shown in the figure.

Fold the sheet of paper in different ways and register the length of *a* and *x*. Analyze your data and find out what is the triangle (T) of larger area formed in the lower left corner of the sheet by the effect of this fold?

(consider a sheet of paper of 29 cm × 21 cm)

Figure 4. Task 2: Folding the corner of a sheet

The task (2) *Folding the corner of a sheet* was assumed as a valuable one. The main characteristic of this task emphasized by the pre-service teachers is the need to collect data. From the point of view of the pre-service teachers this is very important because it stresses the connection between Mathematics and reality.

> In our opinion this task, asking the students to collect data and using the analysis of these data to find the answer, shows how Mathematics has to do with reality. And this is something that is very important to show to the students. Then, this situation corresponds to a polynomial function and so it is something that has everything to do with the Mathematics syllabus.

This group also highlight the close connection to the syllabus allowed by this task, mentioning how it allows the work around a third-degree polynomial function. Another characteristic of this task, assumed as important by this group, is the possibility of using it to introduce the study of third degree polynomial functions or using it to deepen the students' knowledge on this kind of functions.

Slalom was the 3rd task (see Figure 5) included in the set of tasks. This is an open task with a sport context. As so, it can be assumed as an application of mathematics on sports, or as an interdisciplinary approach between mathematics and sports. However, it is mainly a mathematics task, being difficult to assume it has an application of mathematics. In fact, having the skier moving in a parabolic route does not seem to be a very real situation.

(3) Slalom

At a Slalom's competition, the skier must make a route passing between two flags forming a door.

1. Define the calculator window to $x \in [-1, 8]$, $y \in [-1, 7]$. Draw the points (1, 4), (2, 4), (5, 4) , (6, 4), that will represent the Slalom flags.
2. Find a course for the skier, defined by a quadratic function and going through both doors without touching the flags.
3. The public attending the competition occupies the area in the plane defined by $y \geq 6$. Find a new quadratic trajectory for the skier so that he is not crossing the area reserved for the public.
4. The following illustration shows the path made by the Swiss champion who won this Slalom. Find the expression of a piecewise function with this graphic representation.

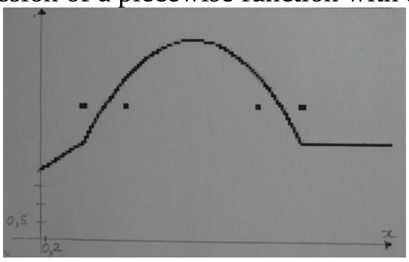


Figure 5. Task 3: Slalom

The task (3) *Slalom* was another of the tasks not chosen by this group of pre-service teachers. According to them and keeping in mind they were asked to choose tasks allowing the students to explore some situations from reality, they found that in this case the reality was somehow unreal. "We think this task is a bit artificial and has nothing to do with the students' reality. Still, the quadratic function can be worked on".

Nevertheless, the pre-service teachers address the fact that the task allows an approach to the quadratic function, one of the contents of the syllabus. They assume the task as a good opportunity for the students to look for suitable functions for the router of the skier, allowing the development of their knowledge on the impact of the parameters on the resulting parabola. As so, they recognize some value on it.

The trains was the 4th task, and it was included in the set of tasks because it has obviously a real context. The task could be addressed in the mathematical field of operations research, however, it does not seem very important to get information about the moment and place where the trains cross. Also, the travelling speed of the trains seems to be a little bit unreal.

(4) The trains

The cities of Lisbon and Oporto are 315 km away from each other.
A train departures from Lisbon to Oporto traveling at an average speed of 50 km/h.
At the same time, another train departs from Oporto to Lisbon traveling at 40 km/h.
How much travel time does it take for the trains to cross?
How far from Lisbon do the trains cross?

(adapted from Neves, Guerreiro, Leite, & Silva, 2010, p. 50)

Figure 6. Task 4: The trains

The task (4) *The trains* (see Figure 6) was one of the tasks chosen by the pre-service teachers. The main reason presented to justify the preference for this task is related to a strong connection to reality. The reasoning required for the students to solve the task is another point addressed and a point that is assumed as an important one. "We think it's a task that has a lot to do with reality. And it is a problem that requires the students to think. We think it's a good task to introduce the linear function".

Despite the relevance ascribed to the reality on the situation presented, the reasonableness of the speeds of the trains considered are not taken into account. It is also not considered the interest of knowing where the trains cross. And even when confronted we these two points, the pre-service teachers keep their point of view unchanged, stating that this task has a strong connection with reality.

Apples was the 5th task (see Figure 7), and it was included in the set of tasks because it starts addressing the important issue of healthy eating. However, the task is about fruit flies. Even so, it can be assumed as an interdisciplinary approach between mathematics and biology, or an application of mathematics on the field of biology, but the model presented is not the best to model population grows or decrease.

(5) Apples

Healthy eating must be rich in fruit. Fruit is rich in vitamins, minerals and phytochemicals that usually have an antioxidant function, among other beneficial effects for health.
But sometimes plagues appear that attack the production of fruit. Last year an apple tree plantation was invaded by a plague of fruit flies. It was found that the number N of fruit flies, in thousands, evolved over time t, in days, according to the following law:

$N(t) = -t^2 + 18t + 40$

a) What was the initial number of fruit flies?
b) After how long has the plague been exterminated?
c) On what day were the largest number of fruit flies detected and what was the number?

(adapted from Neves, Guerreiro, Leite, & Silva, 2010, p. 75)

Figure 7. Task 5: Apples

The task (5) *Apples*, was another of the tasks chosen by the pre-service teachers. In the case of this task, the most important issue for the decision to choose this task is related to the reference to healthy food. This is strongly valued by the pre-service teachers, although this reference is not truly linked to the task. In fact, the task is about a plague of fruit flies. Nevertheless, the initial reference on the task to the relevance of healthy food seems to be relevant enough for these pre-service teachers. They also address the work around the quadratic function as another important element on this task. “We think this task has something very important that is the reference to the issues of healthy food. It alerts for this. And then it allows the students to work on the quadratic function”.

When discussing the connection to reality, it does not seem important to these pre-service teachers that the best model to describe a population increase or decrease is usually an exponential model and not a quadratic one (a potentially relevant point when the intention is to choose tasks to stress the connection between mathematics and reality).

The colony of bacteria was the 6th task. This task is very similar to the previous one, including a model for the number of bacteria in a colony (instead of a model for the number of flies). The main difference is that in this case the task does not address any relevant issue, as it was the case on the previous task.

(6) The colony of bacteria

In a laboratory, a bacterial colony was studied. At eight o'clock it was done the first counting and, after that, a new counting was done every hour. It has been found that the number N of bacteria, in thousands, after h hours, is given by $N (h) = -h^2 + 4h + 9$.

a) How many bacteria did the colony have at 8 o'clock?
b) What was the result of the second counting?
c) At what time of the day did the number of bacteria exceed 9000?
d) Describe the evolution of the colony from 8 to 13 hours.
e) At what time was the colony extinct?

(adapted from Costa & Rodrigues, 2007, p. 118)

Figure 8. Task 6: The colony of bacteria

The similarity between task (6) *The colony of bacteria* and the previous task (the one about apples) is recognized in terms of mathematical content. As so, the pre-service teachers think it does not make sense to choose both. They think the situation on this task is not very appealing to the students and, on the contrary, the previous one addresses healthy food, a very relevant issue. Therefore, they choose task (5) and not this one.

> This task is very similar to that of apples. As so it makes no sense to choose both. It also addresses the quadratic. We chose apples' task for its reference to healthy food. And also, because apples are more interesting than bacteria, are not they?

The box was the 7th task (see Figure 9), and it was included in the set of tasks because it consists on a search for a solution for a situation intending the building of something. Although it is presented as a particular problem, it can be related to economic issues.

Similarly to what happened on tasks (5) and (6), in the case of task (7) *The box*, the first analysis is based on the mathematical content addressed. This task focus on a 3rd degree polynomial function.

(7) The box

Laura intends to build a box without a lid to store her brother's toys. For this she has a rectangular card with 1.2 m long and 80 cm wide, where she intends to remove four square corners to facilitate folding the sides of the box. What is the square side length that Laura should cut at every corner of the card to get a box of maximum volume? (display the results in centimeters, rounding it to two decimal places) Steps to follow in the resolution: - Show that $V(x) = 4x^3 - 4x^2 + 0{,}96x$, being V the volume of the box. - Explain the variation of x and use the graphing calculator to get a graphical representation of the function. - Calculate x so that the volume of the box is maximum. (adapted from Costa & Rodrigues, 2010, p. 93)

Figure 9. Task 7: The box

That is also the case of the task (2) *Folding the corner of a sheet*. As so, the pre-service teachers decided that only one of these tasks should be chosen. The fact that the task (2) address the same content, but requires real data collection, is considered potentially more interesting for the students and it is decisive for the pre-service teachers' choice.

> It's a volume, so it's a 3rd degree polynomial function. And, well, we think the sheet of paper task is more interesting because it includes data collection. We think that it would make it more relevant for the students.

Conclusion

The main conclusions point to an appreciation of the mathematical part of the task and to a devaluation of the remaining components. The focus seems to be on the mathematical content addressed, suggesting a stronger emphasis on the Content Knowledge and a devaluation of the Pedagogic Knowledge. The way how reality appears on the tasks seems not to be assumed as very relevant, suggesting that ACK and APK are not very developed. This result in a choice of tasks where the answers do not make sense in a real context or where the mathematical model advanced on the task is not suitable to model that kind of situation. The interdisciplinary character of the situation (APCK) most of the times seems to be neglected, resulting in a choice of tasks where its presence is poor or resulting in approaches where that component is not explored. These options illustrate a strong appreciation of mathematical procedures and some tendency to approach the contents separately: there seems to be a strong appreciation of disciplinary knowledge, very marked by a traditional approach to the curriculum, where the disciplines and contents of each discipline are approached separately. Opportunities to creates bridges to other disciplines studied by the students, to enlarge the students' culture or to promote their curiosity for new fields are not valued. The usefulness of mathematics in so many real-life situations is not valued either, although it could be used to promote the students' interest in mathematics. And the reason for that seems to be related to the pre-service teachers' knowledge. Understanding the knowledge needed to adopt practices that differ from the more traditional ones is a very relevant matter and somehow not a very well-studied field. As so, it will be important to deepen the understanding about the way how pre-service teachers conceive the integration of applied tasks on the teaching and learning of Mathematics.

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