Instructions and examples for the implementation of approximations for the likelihood ratio statistic for testing the equality of several covariance matrices

The computational modules described are the ones developed in the paper: Marques, F.J. (2018), Products of ratios of gamma functions - An application to the distribution of the test statistic for testing the equality of covariance matrices, Journal of Computational and Applied Mathematics, in press. These modules were developed using the software Mathematica version 10.0 and allow the computation of approximations for the distribution of the likelihood ratio test statistic Λ (and also for its negative logarithm $W = log\Lambda$) used to test the equality of several covariance matrices in the balanced and unbalanced cases. Only one simple module is required for the computation of the probability density and cumulative distribution functions of Λ and W. The module is presented in Figure 1. The use of this module requires the Numerical Calculus package. Before executing this module, it should be used the following instruction `Needs["NumericalCalculus`"]`. To run the function `EqMatrices[Samples_,type_,p_,m_,x_]` in Figure 1 it is necessary to define the following parameters

i) `Samples` - is a vector with the samples size considered for each population;
ii) `type` - can assume two values 1 or 2; if `type=1` the output will be of the probability density function in a given value `x`; if `type=2` the output will be of the cumulative distribution function in a given value `x`;
iii) `p` - the number of variables in each of the populations;
iv) `m` - the number of exact moments matched by the approximating distribution;
v) `x` - the running value.

Figure 1: Module for the approximating probability density and cumulative distribution functions of W

```
EqMatrices[Samples_,type_,p_,m_,x_]:=Module[{q,NN,n,moments1,moments2,b,p1,r1,r2,l1,t,pp,ppp,pe,r1s},
  If[AllTrue[Samples,#>p&]&&IntegerQ[p]&&IntegerQ[m]&&AllTrue[Samples,IntegerQ]&&AnyTrue[{1,2},#==type&],
    ClearSystemCache;q=Length[Samples];NN=Total[Samples]-q;n=Samples-1;
    xkl[k_,l_]:=Sum[n[[i]],{i,1,k-1}]+l;Clear[t];
    moments1=Rationalize[Table[(-1)^(-h)*ND[NN^(NN*t*p/2)/Product[n[[k]]^(n[[k]]*t*p/2),{k,1,q}]*
                            Product[Gamma[(NN+1-j)/2]/Gamma[(NN+1-j)/2+NN/2*t]*
                              Product[Gamma[(n[[k]]+1-j)/2+n[[k]]/2*t]/Gamma[(n[[k]]+1-j)/2],
                                {k,1,q}],{j,1,p}],{t,h},0,Terms->7,WorkingPrecision->250,
                                   Scale->0.00000001],{h,1,If[m<4,4,m]}],0];
    b = Flatten[Table[Table[Table[(NN+1-j)/(2*NN)+(xkl[k,l]-1)/NN-((n[[k]]+1-j)/(2*n[[k]])+(l-1)/n[[k]]),
                  {l,1,n[[k]]}],{k,1,q}],{j,1,p}]];Clear[p1, r1, r2, l1];
    {p1,r1,r2,l1}=Sort[Cases[{p1,r1,r2,l1}/.NSolve[Table[moments1[[h]]==
        Moment[MixtureDistribution[{p1,1-p1},{GammaDistribution[r1,1/l1],GammaDistribution[r2,1/l1]}],h],
           {h,1,4}],{p1,r1,r2,l1}],{_Real,_Real,_Real,_Real}]][[2]];
    If[Abs[r1-Total[b]]<1,r1s=Total[b],r1s=r1];pe=Table[Unique["pe"],{m}];
    moments2=Rationalize[Table[Moment[MixtureDistribution[Flatten[{pe,{1-Total[pe]}}],
               Table[GammaDistribution[r1s+i,1/l1],{i,0,m}]],h],{h,1,m}],0];
    ppp=Rationalize[Flatten[pe/.NSolve[Table[moments2[[h]]==moments1[[h]],{h,1,m}],pe]],0];
    pp=Sum[ppp[[j]],{j,1,Length[ppp]}];l1=Rationalize[l1,0];r1s=Rationalize[r1s,0];
    If[type==1,SetPrecision[Sum[ppp[[j+1]]*PDF[GammaDistribution[r1s+j,1/l1], x],{j,0,m-1}]+
                        (1-pp)*PDF[GammaDistribution[r1s+m,1/l1],x],16],
         If[type==2,SetPrecision[Sum[ppp[[j+1]]*CDF[GammaDistribution[r1s+j,1/l1],x],{j,0,m-1}]+
                            (1-pp)*CDF[GammaDistribution[r1s+m,1/l1],x],16]]],
Print["Incorrect choice of parameters values"]]]
```

For example, let us consider 4 independent samples of sizes $\{20, 30, 10, 20\}$ from 4 multivariate Normal populations, $N_3(\underline{\mu}, \Sigma)$. If one aims to evaluate the approximating cumulative distribution and density functions of W which matches 6 exact moments at the value 2/3, the Mathematica code should be given, respectively, as (please first load the Numerical Calculus Package)

```
SetPrecision[EqMatrices[{20, 30, 10, 20},2,3,6,3/4],12]
SetPrecision[EqMatrices[{20, 30, 10, 20},1,3,6,3/4],12]
```

the results are $4.89888210760 \times 10^{-8}$ and $5.47832786306 \times 10^{-7}$ and are given in about 0.80 seconds in a Personal Computer Intel Core i7 2.00GHz. To compute the probability density or the cumulative distribution functions of Λ, the same module can be used considering the necessary transformations. In the scenario considered in the example given above, the approximating cumulative distribution and density functions of Λ at the same value can be obtained, respectively, in the following way

```
1-SetPrecision[EqMatrices[{20, 30, 10, 20},2,3,6,-Log[3/4]],12]
SetPrecision[EqMatrices[{20, 30, 10, 20},1,3,6,-Log[3/4]]×1/(3/4),12]
```

the results are respectively 0.999999999987 and $5.22293121604 \times 10^{-10}$ and are given in about 0.80 seconds.