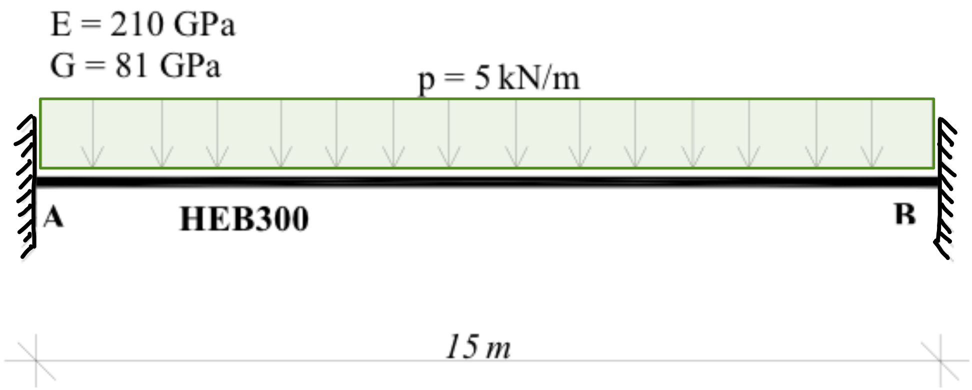
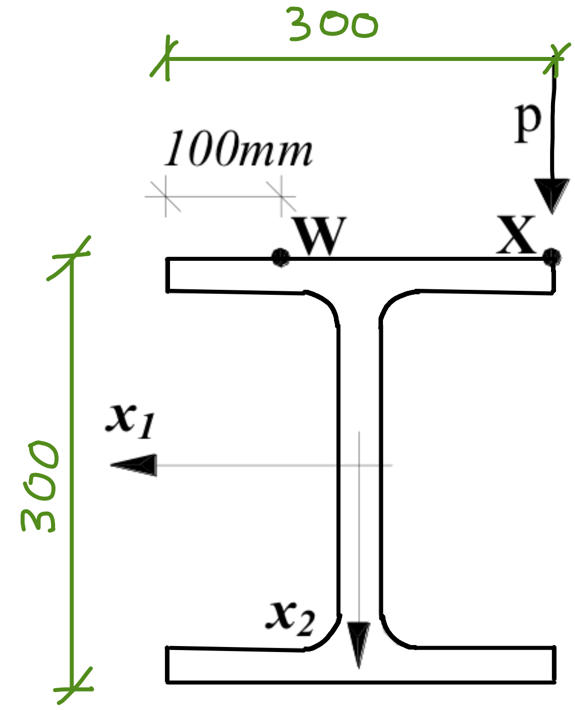
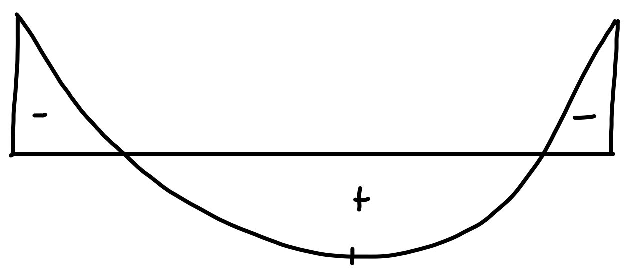




$I_1 = 251,7x10^6\ mm^4$
$t_f = 19\ mm$
$t_w = 11\ mm$

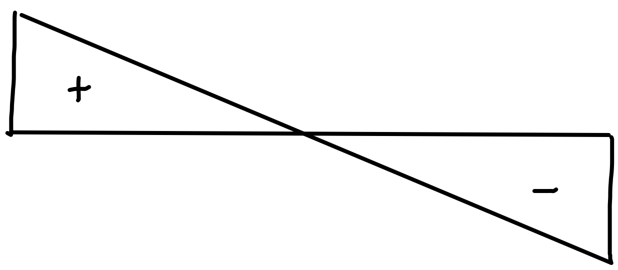
M



$\frac{p\ell^2}{12} = 43,75$ kNm

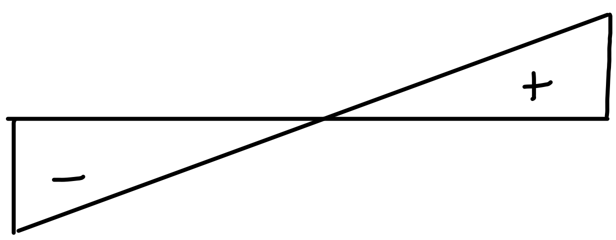
$\frac{p\ell^2}{24} = 46,88$ kNm

V



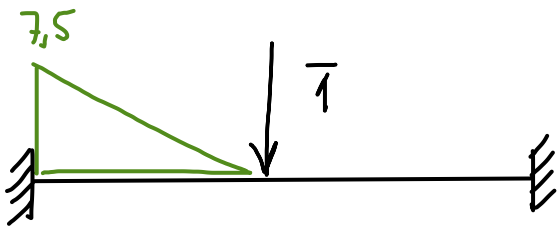
$\frac{p\ell}{2} = \frac{5 \times 15}{2} = 37,5$ kN

T



$\frac{p \times 0,15 \times \ell}{2} = 5,625$ kNm

b. Calcule o deslocamento vertical do centro de gravidade da secção de meio vão.

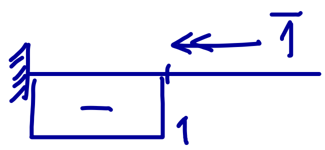


$$\delta = \frac{1}{EI}\int \Big(\underbrace{(+)}_{\frac{pl^2}{8}=140,6} + \overset{93,75}{(-)} \Big) \overset{7,5}{(-)}\ dx + \underbrace{\frac{1}{GA_v}\int \overset{37,5}{(+)}\ \overset{1}{(+)}\ dx}_{\simeq 0 \text{ (desprezar deformação por corte)}}$$

$7,5$ (comprimento)

$$\delta = \frac{7,5}{EI}\left(-\frac{1}{4}140,6 \times 7,5 + \frac{1}{2}93,75 \times 7.5\right) = \underbrace{\frac{659,1}{EI}= 12,48 \text{ mm}}_{\delta = \frac{pl^4}{384EI}}$$

c. Calcule a rotação máxima em torno do eixo da viga, indicando a secção em que se verifica.

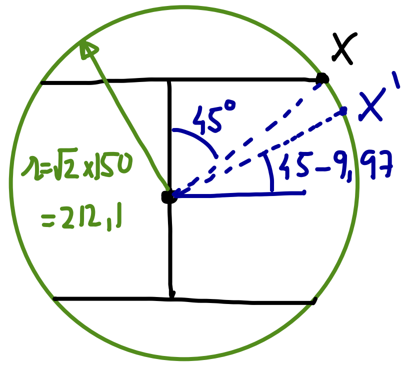


$$J = \frac{1}{3}\left[(0,3-0,019)\times 0,011^3 + 0,6 \times 0,019^3\right] = 1,496 \times 10^{-6} \text{ m}^4$$

$$\varphi = \frac{1}{GJ}\int \overset{5,625}{\triangle}\ \overset{1}{\square}\ dx = \frac{1}{2}\,5,625 \times 7,5\ \frac{1}{81\times 10^6}\ \frac{1}{1,496}\times 10^6 = \underbrace{0,174 \text{ rad}}_{9,97^\circ}$$

7.5 (comprimento)

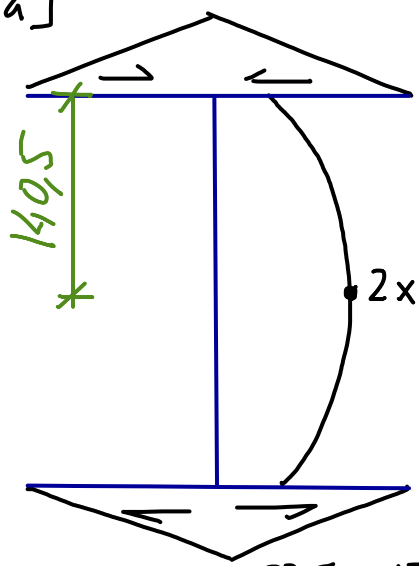
d. Com base nos resultados das alíneas b) e c), determine o deslocamento vertical do ponto **X** da secção de meio vão.



$$\delta_X = 12,48 + \overbrace{\underbrace{0,174 \times 150}_{26,1}}^{\text{H.P.D}} \simeq 38,58 \text{ mm}$$

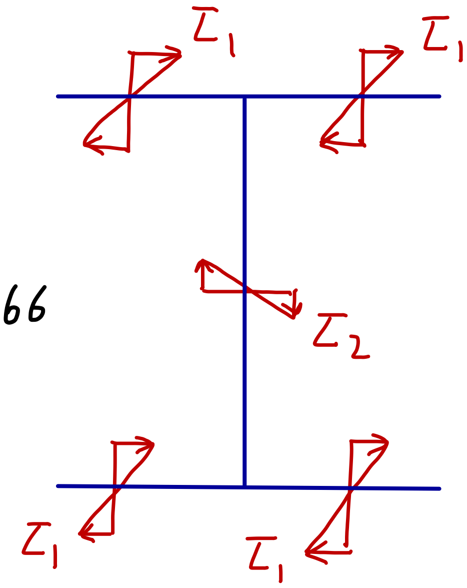
$$\delta_X = 12,48 + \underbrace{212,1\left[\text{sen}45 - \text{sen}(45-9,97)\right]}_{28,43} = 40,71 \text{ mm}$$

[MPa]



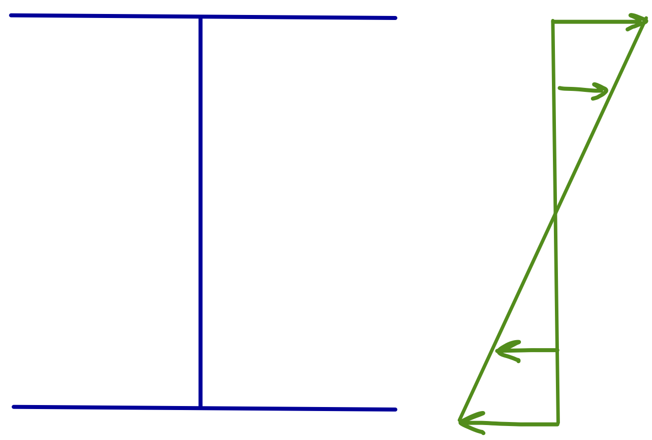
$$2\times 3{,}24\,\frac{19}{11}+\frac{37{,}5\times 0{,}1405^2/2}{251{,}7\times 10^{-3}}=12{,}66$$

$$\frac{37{,}5\times 0{,}15\times 0{,}145}{251{,}7\times 10^{-3}}=3{,}24$$



$$\tau_1=\frac{5{,}625}{1{,}496\times 10^{-6}}\times 0{,}019=71{,}44$$

$$\tau_2=\tau_1\,\frac{0{,}011}{0{,}019}=41{,}36$$



$$\frac{M}{I}\,0{,}15=\frac{93{,}75}{251{,}7\times 10^{6}}\times 0{,}15=55{,}87$$

$$\sigma_{VM}^{W}=\sqrt{\sigma^2+3\tau^2}=\sqrt{55{,}87^2+3(2{,}16+71{,}44)^2}=139{,}2\text{ MPa}$$

$3{,}24$ — 150

τ_w — 100